Dr. I. Gleason **WS 2023/24**
Dr. J. Anschütz

Algebraic Geometry I

13. Exercise sheet

This exercise sheet is designed to be a preparation for the exam and we recommend solving it without consulting the lecture notes steadily.

Exercise 1 (4 Points):

Let k be an algebraically closed field. Find a proper, normal curve $C \subseteq \mathbb{P}^2_k$ with function field isomorphic to $k(x,y)$, $xy^2 = x + 1$.
Hint: Use the Jacobian criterion from sheet 12.

Exercise 2 (4 Points):

1) State the valuative criterion for separatedness.
2) Let $g\colon Z \to Y, f\colon Y \to X$ morphisms of schemes with g proper and surjective, f quasi-separated and $f \circ g$ separated. Show that f is separated.

Exercise 3 (4 points):

Let k be an algebraically closed field. Determine the irreducible components and their intersections for the schematic closure of $V(y^2x + 2yx - y) \subseteq \mathbb{A}^2_k = \operatorname{Spec}(k[x,y])$ in $\mathbb{P}^2_k$ along the embedding $(x,y) \mapsto [x : y : 1]$.

Exercise 4 (4 points):

Let k be an algebraically closed field. Let $f\colon \mathbb{P}^1_k \to \mathbb{P}^1_k, [x : y] \mapsto [x^2 : y^2]$.
1) Show that $f_*(\mathcal{O}_{\mathbb{P}^1_k})$ is a vector bundle $\mathcal{E}$ on $\mathbb{P}^1_k$, which has rank 2.
2) Show that $\mathcal{E} \cong \mathcal{O}_{\mathbb{P}^1_k} \oplus \mathcal{O}_{\mathbb{P}^1_k}(-1)$.

To be handed in on: Thursday, 01.02.2024 (during the lecture, or via eCampus). Please contact your tutor to organize how to receive your corrected exercise sheet.