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WS 2023/24

Algebra II - Local fields

9. Exercise sheet

Exercise 1 (4 points):

Set $G := \mathbb{Z}/2$ with generator σ. Let $M = \mathbb{Z}$ be the G-module with trivial action, and $N = \mathbb{Z}$ with action $\sigma(n) := -n, n \in N$.
1) Find a free resolution $F^\bullet$ of N as a G-module.
2) Calculate $\mathcal{H}^i(\mathrm{Hom}_G(F^\bullet, M))$ and $\mathcal{H}^i(\mathrm{Hom}_G(F^\bullet, N))$ for all $i \geq 0$.

Exercise 2 (4 points):

Let $G = \mathbb{Z}^2$. Calculate $H^i(G, \mathbb{Z})$ for $i \geq 0$, where $\mathbb{Z}$ is given the trivial G-action.

Exercise 3 (4 points):

Let $f^\bullet \colon A^\bullet \to B^\bullet$ be a morphism of complexes of abelian groups. We set $C(f)^i := A^{i+1} \oplus B^i, i \in \mathbb{Z},$, the "mapping cone of $f^\bullet$", with differential $C(f)^i \to C(f)^{i+1},\ (a, b) \mapsto (-d^{i+1}(a), f^{i+1}(a) + d^i b)$.
1) Show that $C(f)^\bullet$ is a complex.
2) Construct a long exact sequence

$$\ldots \to \mathcal{H}^i(A^\bullet) \overset{\mathcal{H}^i(f)}{\to} \mathcal{H}^i(B^\bullet) \to \mathcal{H}^i(C(f)^\bullet) \to \mathcal{H}^{i+1}(A^\bullet) \to \ldots$$

Exercise 4 (4 points):

Consider the diagram of abelian groups

$$\begin{array}{ccccccccc} M_1 & \longrightarrow & M_2 & \longrightarrow & M_3 & \longrightarrow & M_4 & \longrightarrow & M_5 \\ \downarrow f_1 & & \cong \downarrow f_2 & & \downarrow f_3 & & \cong \downarrow f_4 & & \downarrow f_5 \\ N_1 & \longrightarrow & N_2 & \longrightarrow & N_3 & \longrightarrow & N_4 & \longrightarrow & N_5 \end{array}$$

with exact rows and f_2, f_4 isomorphisms.
1) Assume f_1 is surjective. Show f_3 is injective.
2) Assume f_5 is injective. Show f_3 is surjective.

To be handed in on: Thursday, 14.12.2023 (during the lecture, or via eCampus).