Algebra II - Local fields

4. Exercise sheet

Exercise 1 (4 points):

Prove that $\mathbb{C}_p := \widehat{\overline{\mathbb{Q}_p}}$ is algebraically closed.

Exercise 2 (4 points):

Set $K := \mathbb{Q}(i)$. Find a prime p and two inequivalent extensions of the p-adic valuation on $\mathbb{Q}$ to K.

Exercise 3 (4 points):

Let $K := \mathbb{Q}_3(\alpha)$ with $\alpha^6 - 12\alpha^3 + 360$, and let $|-|$ be the extension of the p-adic norm to K.
1) Show that $\mathbb{Q}_3(\sqrt{2}) \cong \mathbb{Q}_3(\sqrt{-1})$ is a subfield of K.
2) Determine the degree $[K : \mathbb{Q}_3]$, the ramification index $e(K/\mathbb{Q}_3)$, the inertia degree $f(K/\mathbb{Q}_3)$ and the norm $|\alpha|$.

Exercise 4 (4 points):

Let $n \geq 1$ and let $K/\mathbb{Q}_p$ be a finite extension. Show that K has (up to isomorphism) only finitely many field extensions of degree $\leq n$.
Hint: Use Krasner's lemma and the fact that Eisenstein polynomials are irreducible.

To be handed in on: Thursday, 09.11.2023 (during the lecture, or via eCampus).