$\mathbb{B}_C$ = closed unit ball over C, C (alg. cld) non-arch. field of char p

$= \mathrm{Spa}(C\langle T\rangle, \mathcal{O}_C\langle T\rangle)$

If $E = \mathbb{F}_q((\pi))$, $\mathbb{F}_q \hookrightarrow C$

$\Rightarrow$ $\mathrm{Spa}(C, \mathcal{O}_C) \times_{\mathrm{Spa}\,\mathbb{F}_q} \mathrm{Spa}(\mathbb{F}_q((\pi))) = \mathbb{D}_C^* =$ punctured open unit disc over C $\subseteq \mathbb{B}_C$

Def ($E = \mathbb{F}_q((\pi))$)

$X^{ad}_{E,C} := \mathbb{D}_C^* / \varphi_C^{\mathbb{Z}}$

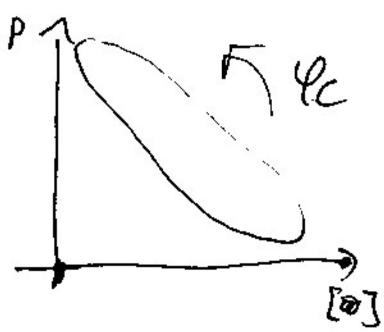
$$\begin{array}{c} Y^{ad}_{E,C} = \mathbb{D}_C^* \subseteq \mathbb{B}_C \\ \downarrow \\ X^{ad}_{E,C} = Y^{ad}_{E,C} / \varphi_C^{\mathbb{Z}} \end{array}$$

free, properly disc.

If $E = \mathbb{Q}_p$

$Y^{ad}_{\mathbb{Q}_p,C} = \mathrm{Spa}(W(\mathcal{O}_C)) \setminus V(p \cdot [\varpi])$, $\varpi \in C$, $0 < |\varpi| < 1$

Picture:



(Dictionary

$E = \mathbb{F}_q((\pi))$	$E = \mathbb{Q}_p$
$\mathcal{O}_C[[\pi]]$	$W(\mathcal{O}_C)$
$Y^{ad}_{E,C} = \mathbb{D}_C^*$	$Y^{ad}_{E,C}$
$\mathbb{B}_C$	??

$X^{ad}_{\mathbb{Q}_p,C} = Y^{ad}_{\mathbb{Q}_p,C} / \varphi_C^{\mathbb{Z}}$

$\underline{E} = \mathbb{Q}_p$

$X_{\mathbb{Q}_p, C}$ scheme, Dedekind $\leadsto$ justifies "the curve"

$\downarrow$ $\leadsto$ ~~loc. of~~ not loc. of finite type, $H^0(X, \mathcal{O}) = \mathbb{Q}_p$

$\operatorname{Spec} \mathbb{Q}_p$

but residue fields at closed point are untilts of C, e.g. $\mathbb{C}_p = \widehat{\overline{\mathbb{Q}}_p}$

$\Omega^1_{X/\mathbb{Q}}$ horrible

~~Lecture~~ No duality

$$H^i(X, \mathcal{F}) \times H^{1-i}(X, \mathcal{F}^\vee \otimes \omega) \to \mathbb{Q}$$

for vectorbundles $\mathcal{F}$ on X

Later: Usually $\dim H^i(X, \mathcal{F}) = \infty$

Fact: $H^1(X, \mathcal{O}) = 0$ ($\Rightarrow$ X "behaves" a bit like $\mathbb{P}^1$)

Pick $x \in X$ closed point

$\leadsto \quad 0 \to \mathcal{O} \to \mathcal{O}(x) \to i_{x*} k(x) \to 0$, $\quad k(x) = \mathbb{C}_p$ e.g.

take H^i

$\leadsto \quad 0 \to \mathbb{Q}_p \to H^1(X, \mathcal{O}(x)) \to \mathbb{C}_p \to 0$

$H^1(X, \mathcal{O}(x))$: inf. dim.

$\mathbb{B}_C$

~~Rem~~ $\mathcal{O}_C[[\pi]] \supseteq \mathcal{O}_C\langle\pi\rangle = \mathcal{O}_C[\pi]^\wedge_{\varpi}$ $(\sim)$ defines $\mathbb{B}_C$ ~~Spa~~

$$W(\mathcal{O}_C) = \left\{ \sum_{i=0}^{\infty} [x_i] p^i \;\middle|\; x_i \in \mathcal{O}_C \right\} \supseteq \underbrace{\left\{ \sum_{i=0}^{n} [x_i] p^i \;\middle|\; x_i \in \mathcal{O}_C, n \in \mathbb{N} \right\}}_{\text{not a subring!}}$$

$$\left\{ \sum_{i=0}^{n} [x_i] p^i \;\middle|\; x_i \in \mathcal{O}_C, n \in \mathbb{N} \right\}$$

$$|\operatorname{Spa}(\mathbb{Q}_p, \mathbb{Z}_p)| = 1$$

$$\# \operatorname{Spa}(\mathbb{F}_q((\pi)), \mathbb{F}_q[[\pi]]) = 1 = \# \operatorname{Spa}(\mathbb{F}_q((\pi^{\frac{1}{p^\infty}})), \mathbb{F}_p[[\pi^{\frac{1}{p^\infty}}]])$$

$$v(x^p) = v(x)^p$$

~~Spec~~

$$\operatorname{Spec} B \times_{\operatorname{Spec} A} \operatorname{Spec} C \simeq \operatorname{Spec}(B \otimes_A C)$$

This is not true for adic spaces:

$$\operatorname{Spa}(C, \mathcal{O}_C) \times_{\operatorname{Spa}(\mathbb{F}_q)} \operatorname{Spa}(\mathbb{F}_q((\pi))) \simeq \underbrace{\mathbb{D}_C^*}_{\text{not q.c.}}$$

Problem: $\operatorname{Spa}(C, \mathcal{O}_C) \to \operatorname{Spa}(\mathbb{F}_q)$ are not adic

$\operatorname{Spa}(\mathbb{F}_q((t))) \to \operatorname{Spa}(\mathbb{F}_q)$

$\# \, \mathrm{Spa}(C, C^+) =$ Krull dimension of C^+

$C^+ \subseteq C$ open + bdd val. subring

$$
\begin{tikzcd}
\mathrm{Spa}(\mathcal{O}_C[[\pi]]) \setminus V(\pi, [\varpi]) \arrow[d] \arrow[dr, bend left] & \\
\mathrm{Spa}(\mathcal{O}_C) & \mathrm{Spa}(C, \mathcal{O}_C) \arrow[l]
\end{tikzcd}
$$

$$= \mathrm{Spa}(\mathcal{O}_C) \setminus V(\varpi)$$