Dr. A. Mihatsch
Dr. J. Anschütz **SS 2023**

Algebra I - Commutative algebra

12. Exercise sheet

This is the last exercise sheet for this semester.

Exercise 1 (4 points):
Let A be ring and let G be a finite group acting on A by ring automorphisms. Let A^G be the ring of invariants of G in A.
i) Show that A is integral over A^G.
Hint: For $a \in A$ consider the polynomial $\prod_{g \in G}(T - g(a))$.
ii) Assume that A is a domain with quotient field K. Show that $K^G = \mathrm{Quot}(A^G)$.

Exercise 2 (4 points):
Let A be a normal domain with quotient field K and let G be a finite group acting on A by ring automorphisms.
i) Show that A^G is normal.
ii) Let k be a field of characteristic $\neq 2$. Show that $k[x, y, z]/(z^2 - xy)$ is normal.
Hint: Sheet 9, exercise 3.

Exercise 3 (4 points):
Let L/K be a finite Galois extension of number fields with Galois group G. Show that $\mathcal{O}_L$ is stable under the action of G on L and that $\mathcal{O}_L^G = \mathcal{O}_K$.

Exercise 4 (4 points):
Let k be a field and let $A := k[x, y]/(y^2 - x^3 - x^2)$.
i) Show that A is a domain.
ii) Show that $t := y/x \in \mathrm{Quot}(A)$ does not lie in A.
iii) Show that t is integral over A.
iv) Show that $\mathrm{Quot}(A) = k(t)$ and that $k[t] \subseteq \mathrm{Quot}(A)$ is the normalization of A.

To be handed in on: Thursday, 06.07.2023 (during the lecture, or via eCampus).