Exercise 1 (4 points):
Let \( f: Y \to X \) be a morphism of schemes and let \( F \) be an abelian sheaf on \( Y \). Prove that the canonical morphism

\[
H^n(X, f_* (F)) \to H^n(Y, F)
\]

is an isomorphism for any \( n \geq 0 \) if

i) \( f \) is a closed immersion, or

ii) \( f \) is affine and \( F \) is a quasi-coherent \( \mathcal{O}_Y \)-module.

Exercise 2 (4 points):
Let \( X \) be a topological space and let \( F \) be a sheaf of abelian groups on \( X \). Let \( X = U \cup V \) be an open covering of \( X \). Prove that there exists a (natural) long exact Mayer-Vietoris sequence

\[
\ldots \to H^i(X, F) \to H^i(U, F) \oplus H^i(V, F) \to H^i(U \cap V, F) \to H^{i+1}(X, F) \to \ldots.
\]

Hint: Analyze a suitable spectral sequence.

Exercise 3 (4 points):
Let \( A \) be a ring and let \( d < 0 \) (the case \( d \geq 0 \) has been handled in the lecture). Prove that

\[
H^i(P^n_A, \mathcal{O}_{P^n_A}(d)) \cong \begin{cases} 0 & \text{if } i < n \\ \left( \frac{1}{x_0 \ldots x_n} A[x_0^{-1}, \ldots, x_n^{-1}] \right)_d & \text{if } i = n \end{cases}
\]

for every \( n, i \geq 0 \). Here the subscript \( d \) denotes the space of homogenous polynomials of degree \( d \).

Thus in particular, \( H^i(P^n_A, \mathcal{O}_{P^n_A}(d)) = 0 \) if \(-n - 1 < d < 0\).

Hint: Use Čech cohomology for the standard covering of \( P^n_A \). To prove the vanishing statement use induction on \( n \) and the short exact sequence

\[
0 \to \mathcal{O}_{P^n_A}(d-1) \xrightarrow{x_0} \mathcal{O}_{P^n_A}(d) \to i_* (\mathcal{O}_{P^{n-1}_A}(d)) \to 0
\]

where \( i: P^{n-1}_A \to P^n_A, (x_1: \ldots: x_n) \mapsto (0: x_1: \ldots: x_n) \).

Exercise 4 (4 points):
Show that

\[
\dim_k H^i(P^n_k, A^i \Omega^{1}_{P^n_k/k}) = \begin{cases} 1 & \text{if } i = j \leq n \\ 0 & \text{otherwise} \end{cases}
\]

and try to find explicit generators. Deduce that \( \Omega^{1}_{P^n_k/k} \) is not an extension of line bundles if \( n \geq 2 \).

Hint: Use the Euler sequence from Exercise sheet 7, Exercise 1.

To be handed in on: Monday, 26. June 2017.