

Seminar ‘Advanced Hodge theory and the Hodge conjecture’

Summer term 2012

The idea of the seminar is to learn some classical concepts and recent approaches to the Hodge conjecture. Basic concepts of Hodge theory are assumed, but we will have the time to recall some of the more technical notions. For further information or if you want to give a talk in the seminar, please contact me at huybrech@.. The **first talk will be on April 10.**

The following is open for discussion. We may decide to shift the emphasize or dwell longer on certain aspects. Personally, I would be more interested in the geometric picture than in the technical details.

10 April: Introduction and survey. Hodge conjecture, generalized Hodge conjecture, variational Hodge conjecture (Speaker: Daniel Huybrechts)

17 April: Absolute Hodge classes and Deligne’s Principle B. Absolute Hodge classes, global invariant cycle theorem,... See [6, 8, 17, 16]. (Maybe restrict to the de Rham aspect of the notion.) Roughly, a class is absolute Hodge if it stays rational under base change by an automorphism of the base field. The Hodge conjecture predicts that all Hodge classes are absolute. (Speaker: Daniel Huybrechts)

24 April: Nodes and the Hodge conjecture. This is [14]. (Speaker: Stefanie Anschlag)

8 May: Hodge conjecture for varieties over $\bar{\mathbb{Q}}$. Under additional assumptions it was shown by Voisin in [16, Sect. 2] that the Hodge conjecture for varieties defined over $\bar{\mathbb{Q}}$ would imply the full Hodge conjecture. (Speaker: Ziyu Zhang)

15 May: Hodge loci are algebraic. This is the famous theorem of Cattani–Deligne–Kaplan. See also the extension to MHS in [2]. We will only have the time for an outline of the proof following [16]. (Speaker: Timo Schürg)

5 June: Preparations: Intermediate Jacobians and normal functions.

Here we should recall standard notions, like intermediate Jacobian, Abel–Jacobi map (see e.g. [3, 9, 17]), etc. Everything put in families leads to normal functions (for homologically trivial cycles). (Speaker: Marti Lahoz)

12 June: Zero loci of normal functions. This is the analogue of the CDK theorem for zero loci of normal functions due to Brosnan, Pearlstein, Schnell [1]. The arithmetic aspect is discussed in [5]. Again, this can only be a survey. (Speaker: Sven Meinhardt)

26 June: Mumford-Tate groups and Noether-Lefschetz loci. Based on Chapter II.C of [11]. (Speaker: Daniel Greb)

10 July: Singularities of normal functions and the Hodge conjecture. In [10] is shown that the Hodge conjecture is equivalent to the existence of singularities of normal functions. We should try to see what this means in concrete terms. (Speaker: Stefanie Anschlag)

References

- [1] P. Brosnan, G. Pearlstein, *Zero loci of admissible normal functions with torsion singularities*. Duke Math. J. 150 (2009), 77–100. arXiv:0803.3365.
- [2] P. Brosnan, G. Pearlstein, Ch. Schnell, *The locus of Hodge classes in an admissible variation of mixed Hodge structure*. arXiv:1002.4422v1.
- [3] J. Carlsson, S. Müller-Stach, C. Peters, *Period mappings ad period domains*. Cambridge studies in advanced mathematics. 2003.
- [4] E. Cattani, P. Deligne, A. Kaplan *On the locus of Hodge classes*. J. AMS 8 (1995), 483–506.
- [5] F. Charles *On the zero locus of normal functions and the étale Abel–Jacobi map*. IMRN 12 (2010) 2283–2304.
- [6] F. Charles, Ch. Schnell *Notes on absolute Hodge classes*. arXiv:1101.3647
- [7] M. de Cataldo, L. Migliorini *On singularities of primitive cohomology classes*. Proc. AMS 137 (2009), 3593–3600. arXiv:0711.1307v1
- [8] P. Deligne *Hodge cycles on abelian varieties*. (notes by J. S. Milne), Lecture Notes in Mathematics, vol. 900 (Springer, Berlin, 1982), 9–100.
- [9] M. Green *Infinitesimal methods in Hodge theory*. In Lecture Notes 1594: Algebraic cycles and Hodge theory.
- [10] M. Green, P. Griffiths *Algebraic cycles and singularities of normal functions*. in Algebraic Cycles and Motives, Vol. 1 (J. Nagel and C. Peters, eds) LMS. Lect. Note Series 343, CUP (2007), 206–263.
- [11] M. Green, P. Griffiths, M. Kerr *Mumford-Tate Groups and Domains: Their Geometry and Arithmetic*. Annals of Math. Stud. Princeton.
- [12] P. Griffiths *Topics in Transcendental Algebraic Geometry*. Annals of Math. Studies 106 (1984), Princeton Univ. Press, Princeton, NJ.
- [13] M. Saito *Generalized Thomas hyperplane sections and relations between vanishing cycles*. arXiv:0806.1461.
- [14] R. Thomas *Nodes and the Hodge conjecture*. J. Alg. Geom. 14, (2005), 177–185.
- [15] C. Voisin *Hodge loci*. Handbook of moduli.
- [16] C. Voisin *Hodge loci and absolute Hodge classes*. Comp. Math. 143 (2007), 945–958.
- [17] C. Voisin *Hodge Theory and Complex Algebraic Geometry I, II* Cambridge studies in advanced Mathematics 76, 77, CUP (2002, 2003).