Graduate Seminar on Algebra (S4A1)

Cubic hypersurfaces (Summer term 2020)

Algebraic geometry really starts with cubic hypersurfaces, i.e. zero sets of one polynomial of degree three (linear and quadratic polynomials belong to linear algebra). Examples include elliptic curves and cubic surfaces. Their theories are both classical subjects with a long tradition. But things get even more interesting in higher-dimensions.

The goal of the seminar is to learn some standard techniques in algebraic and complex geometry (intersection theory, Chow groups, Hodge structures, motivic constructions, deformation theory, moduli spaces, etc.) and to apply them to non-trivial examples all related to cubic hypersurfaces. Most of the talks will be based on material in the lecture notes [Cubics] and the references therein.

Background material will be provided whenever needed (depending on the knowledge of the participants) and I will frequently give quick introductions into some general bits of algebraic geometry and more advanced material. Inevitably certain foundational material will be used as black boxes (learning by doing!).

Prerequisite: Algebraic geometry (varieties, schemes, sheaves, cohomology, curves, basic constructions). Some basic understanding of singular cohomology, Chern classes and Hodge theory (we may build in some refreshers on the latter).

Registration: Please contact me as early as possible if you are interested in participating. Email to `huybrech@...` Please describe you background in algebraic and complex geometry and state your preferences for possible talks.

Time: Thursday 4:15 - 6:00 pm, SR 0.006

List of talks: The list is preliminary and incomplete. Also the order may change.

1. **Hypersurfaces:** Standard material: cohomology, normal bundle sequence, canonical bundle, numerical invariants (Section 1.1.1-1.1.4)
2. **Quadric fibrations** TBC (Based on Section 1.4)
3. **Lines in cubic hypersurfaces:** Lines of the first and second type. Existence of lines. Projection from linear subspaces, (uni)rationality. (Section 3.1.3-3.1.4)
4. **Fano scheme of lines:** Grassmannian, universal Fano scheme of lines, smoothness, deformation theory, Hilbert schemes, canonical bundle, connectedness (Section 3.1.1 and parts of 3.1.2, 3.2.1)
5. **The motive of the Fano scheme:** Relate the Fano variety of lines to the symmetric product. Conclude numerical and cohomological information about the Fano variety. (Section 3.3.1. Background on $K_0(Var)$ and $K_0(Mot)$ will be provided).

6. **Cohomology of the Fano scheme:** Here some of the general results will be spelled out in terms of Hodge structures and in small dimensions. In particular, classical things like the existence of 27 lines on any cubic surface or the connectedness of the Fano variety of lines just fall out of the general machinery. (Section 3.3.3)
7. **Fano correspondence:** There is a more direct way to relate cubic and its Fano variety via the Fano correspondence. (Section 3.4.1)
8. **Linear systems:** TBC (Material from Section 1.2-1.3)
9. **Cubic threefolds I:** TBC (Material from Chapter 5)
10. **Cubic threefolds II:** TBC (Material from Chapter 5)
11. **Cubic fourfolds I:** TBC (Material from Chapter 6)
12. **Cubic fourfolds II:** TBC (Material from Chapter 6)

[Cubics] D. Huybrechts *The geometry of cubic hypersurfaces*, `http://www.math.uni-bonn.de/people/huybrech/Notes.pdf`