Prof. Dr. Matthias Lesch
Mathematisches Institut
Universität Bonn

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Atiyah-Singer Index Theory I
Set 13

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Exercise 1. Show that the symbol map $\sigma\colon CL(V,q) \to \Lambda^*(V)$ and the quantization map $c\colon \Lambda^*(V) \to CL(V,q)$ defined in the lecture are (well-defined) inverses of each other.

Exercise 2. Show that $CL^2(\mathbb{R}^n) = c(\Lambda^2\mathbb{R}^n)$ is a Lie-subalgebra of $CL(\mathbb{R}^n)$ isomorphic to $so(n)$ via the map

$$\tau\colon CL^2(\mathbb{R}^n) \to so(n) \subset \mathrm{End}(\mathbb{R}^n)$$

given by $\tau(a)v = [a,v]$. Deduce that τ exponentiates to a representation

$$\tau\colon \mathrm{Spin}(n) \to SO(n)$$

given by $\tau(g)v = gvg^{-1}$.

Exercise 3. Identify Spin(4).