Prof. Dr. Matthias Lesch
Mathematisches Institut
Universität Bonn

Winter semester 2006/2007

Atiyah-Singer Index Theory I
Set 1

23.10.2006

Exercise 1. Read and understand the proof of the Gelfand-Naimark theorem (for example in *Higson/Roe: Analytic K-Homology* or *Hirzebruch/Scharlau: Einführung in die Funktionalanalysis*).

Exercise 2. Check:

(i) Let X be a compact Hausdorff space. Then $C(X)$ with $\|f\|_\infty := \sup_{x\in X} |f(x)|$ and $f^*(x) := \overline{f(x)}$ is a C^*-algebra.

(ii) Let H be a Hilbert space. Then the bounded linear operators $B(H)$ with $\|T\| := \sup_{\|x\|=1} \|T(x)\|$ and T^* the adjoint operator is a C^*-algebra.

Exercise 3. Let $\mathscr{A} = C(X)$ be a commutative and unital C^*-algebra. If Z is a closed subset of X, then the set of those functions in $\mathscr{A}$ whose restriction to Z is zero constitutes an ideal in $\mathscr{A}$. Prove that every ideal arises in this way.