RESEARCH STATEMENT

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My main research area Chern-Simons gauge theory is a part of topology, where knot theory, differential geometry, global analysis and mathematical physics meet. Recently, I have also made some fascinating discoveries in my interdisciplinary research involving music theory, psychoacoustic neuroscience, and geometry. Some aspects of the above as well as (topological) quantum computing give excellent research topics for undergraduates.

In 1989, Witten [51] took the mathematical community by storm with Chern-Simons theory. It was a beautiful idea, which created entire new lines of research and represented one of the milestones in topology. In physics terminology, this is a quantum field theory in three dimensions, whose action is proportional to the Chern-Simons function

$$\mathrm{CS}(A) = \frac{1}{8\pi^2}\int_M \mathrm{tr}(A \wedge dA + \frac{2}{3}A \wedge A \wedge A).$$

Here $A \in \Omega^1(M;\mathfrak{g})$ is a $\mathfrak{g}$–valued 1–form on an oriented 3–manifold M, where $\mathfrak{g}$ is the Lie algebra of a matrix Lie group G. In more mathematical terms, this theory is an oscillatory integral over $\Omega^1(M;\mathfrak{g})$ with $2\pi ki \cdot \mathrm{CS}$ as a phase function for some integer k. Witten interpreted this as a topological quantum field theory (TQFT) as well as via an asymptotic expansion as $k \to \infty$ known as the perturbative expansion, in which the leading term is given by the method of stationary phase. Even though the infinite-dimensional integral does not have a mathematically rigorous meaning, both interpretations of it can be treated rigorously, and we expect the following. The TQFT should have an expansion (asymptotic expansion conjecture) of a certain form, and this expansion should be equal to the perturbative expansion (perturbative expansion conjecture) [4, Section 7.2].

There have been quite a few advances in understanding the two approaches to Chern-Simons theory separately, notably the first rigorous construction of the TQFT [47], the first computer calculations [27], the construction of the TQFT using skein theory [13, 14, 15], work on the asymptotic expansion conjecture [35, 36, 37, 38], the geometric construction and its asymptotic properties [5, 7, 9, 10], work on the perturbative expansion for $\mathbf{R}^3$ [24, 29, 30, 31, 32, 33, 34] and for general 3–manifolds [11, 12].

My research focuses on the perturbative expansion conjecture, which expresses the asymptotic expansion of the TQFT as $k \to \infty$ in terms of an integral of a combination of classical topological invariants of the conjugacy classes of representations of the fundamental group. Details can be found in the appendix. There have been very few developments apart from the direct comparison for lens spaces for $G = \mathrm{SU}(2)$ and some torus bundles over S^1 by Jeffrey [43], for which I developed some technical machinery for spectral flow computations [40, 41] in order to confirm [43, Conjecture 5.8]. The terms in the asymptotic expansion are conjectured to be well-known classical topological invariants like Reidemeister torsion and the Rho invariant. Jørgen Ellegaard Andersen (Aarhus University) and I have recently identified the leading order term for finite order mapping tori [8] in surprising generality. Together with Brendan McLellan (Aarhus University) and Søren Fuglede Jørgensen (Aarhus University) we are working on identifying lower order terms and on generalizing the results to certain Seifert fibered manifolds as well as certain links in such manifolds. By looking at links in 3–manifolds we naturally encounter the problem of defining and studying Reidemeister torsion and the Rho invariant for manifolds with boundary, which I would like to explore. Furthermore, Atle Hahn (Lisbon University) and I are pursuing a rigorous approach using discrete differential geometry in the spirit of [1, 2].

Among the topological invariants which appear in the asymptotic expansion we find the SU(2) Casson invariant [3]. If M is a homology 3–sphere, that is, a closed 3–manifold with the same integral cohomology as the 3–sphere, then it is an algebraic count of SU(2)–representations of the fundamental group of M. Taubes [49] showed that it has a gauge theoretic interpretation as an Euler characteristic on a quotient of $\Omega^1(M;\mathfrak{su}(2))$ in the spirit of the Poincaré-Hopf theorem by treating the Chern-Simons function as a Morse function. By viewing the critical points of the Chern-Simons function as a Morse complex, Floer extended this idea to his influential instanton Floer homology [26], which has the SU(2) Casson invariant as its Euler characteristic. The Casson invariant has been generalized to other 3–manifolds and other structure groups. In particular, there have been generalizations to SU(3) by Boden, Herald and Kirk [18] as well as to $SL(2,\mathbf{C})$ by Curtis [25]. Details can be found in the appendix.

Paul Kirk (Indiana University, Bloomington), Matthias Lesch (Bonn University) and I have proven the existence and continuity for the Calderón projector for the Hessian of the perturbed Chern-Simons function on a 3–manifold with boundary [42]. This is a key result for computations of the SU(3) Casson invariant, when we need to perturb the Chern-Simons function. Hans Boden (McMaster University, Hamilton) and I have identified the SU(3) Casson invariant of the spliced sum of $(2,p)$ and $(2,q)$ torus knots with 16 times the product of their SU(2) Casson knot invariants [22]. Together with Chris Herald (University of Nevada, Reno), we are in the process of writing up an extension of this result to spliced sums of two arbitrary torus knots, where we need to perturb the Chern-Simons function. Starting with ideas from [39], Andriy Haydys (Imperial College London) and I are working on a new approach to the $SL(2,\mathbf{C})$ Casson invariant by compactifying the representation variety using stratified Morse theory and also on giving a gauge theoretic interpretation. Recently, Peter Kronheimer and Tom Mrowka have revived the interest in instanton Floer homology for knots by discovering connections to the Alexander polynomial and to Khovanov homology [45, 44]. Raphael Zentner (University of Cologne) and I are are working on computing the instanton Floer homology for torus knots.

My interdisciplinary research with music is an attempt to answer the psychoacoustic question posed by Tymoczko, how his geometry of chords [50] relates to perceptual judgments of chord similarity. I am working on defining some characteristic functions on the space of chords, which describe perceptive sound qualities of chords and might explain some music theoretical principles about chord progressions. For example the notion of tension is freely used and vaguely explained in the music theory literature. Usually tension refers to the added notes in a chord, which create tension and can be resolved. On the other hand there are several theories for dissonance in the literature, which is often used as a neuroscientific explanation for tension. David Huron lists 14 of these theories and classifies them as acoustic, psychophysical, cognitive or enculturated. They do not all attempt to describe the same phenomenon. While some correspond to a roughness in sound, others go back to Galilei and correspond to a certain irregularity in the sound. I can show that these are fundamentally different phenomena, which are unfortunately seldom separated in the literature. The meaning of dissonance is further watered down by empirical results on the perceived pleasantness of intervals and chords. Some rigorous mathematical work is necessary to put all of this in order. I have a reasonable mathematical definition for tension, which corresponds very well with the perceived tension of intervals and explains some mysterious phenomena, which to date neuroscientists and music theorists do not have a satisfying explanation for, notably why the chord C-B sounds more dissonant than C-E-G-B [46]. I will present these discoveries at the forthcoming AMS joint meeting in Boston.

Furthermore, Jørgen Ellegaard Andersen, Nicolai Reshetikhin and I want to eventually extend my lecture notes to the Ph.D. course *Lie groups and Chern-Simons theory* given at Aarhus University in the Fall of 2009 to provide a detailed and rigorous introduction to the field.

The following appendix contains additional details and an abbreviated reference list.

Appendix

The perturbative expansion conjecture. Witten [51] described an invariant of a 3–manifold M for each integer level $k \in \mathbf{Z}$ as the (non-rigorous) Feynman path integral

$$Z_k(M, L) = \int_{\mathcal{A}} e^{2\pi k i\ \mathrm{CS}(A)} dA,$$

where $\mathcal{A}$ is the space of G–connections on M, which can be identified with $\Omega^1(M, \mathfrak{g})$ for a trivialization of the principal G–bundle over M. These invariants can be viewed as a TQFT, which is a functor from a certain category of cobordisms to the category of vector spaces. Based on this interpretation, Reshetikhin and Turaev gave a rigorous definition for $G = \mathrm{SU}(2)$, which was later extended to other groups.

For a function $f\colon \mathbf{R}^n \to \mathbf{R}$ with finitely many non-degenerate critical points and a compactly supported function $\varphi\colon \mathbf{R}^n \to \mathbf{R}$, we have the asymptotic behavior

$$\int_{\mathbf{R}^n} e^{ikf(x)} \varphi(x)\, dx \sim_{k\to\infty} \left(\frac{2\pi}{k}\right)^{\frac{n}{2}} \sum_{x \in \mathrm{Crit}(f)} e^{\frac{\pi i}{4} \operatorname{sign} \mathrm{Hess}_x(f)} \frac{e^{ikf(x)}\varphi(x)}{\sqrt{|\det \mathrm{Hess}_x(f)|}}$$

by the method of stationary phase. We may assume that $\varphi(x) = 1$ for $x \in \mathrm{Crit}(f)$ and $\varphi \equiv 0$ outside of a compact set. Therefore, we will abuse the notation and eliminate the function φ from the formulas entirely.

Global analysis provides rigorous generalizations of the determinant and the signature to a formally self-adjoint elliptic differential operator D acting on sections of a vector bundle over a closed manifold M. We can define

$$\det D = e^{-\zeta_k'(0)}, \text{ where } \zeta(s) = \sum_{0 \neq \lambda \in \mathrm{Spec}(D)} \lambda^{-s}$$

and

$$\eta(D, s) := \sum_{0 \neq \lambda \in \mathrm{Spec}(D)} \frac{\mathrm{sgn}(\lambda)}{|\lambda|^s}, \quad \mathrm{Re}(s) \text{ large.}$$

The function $\eta(D, s)$ admits a meromorphic continuation to the whole s-plane with no pole at the origin. Then $\eta(D) := \eta(D, 0)$ is called the η–invariant of D and generalizes the signature.

Up to an operator with symmetric spectrum, the Hessian of the Chern-Simons function at a critical point A is equal to the odd signature operator coupled to A, which is the formally self-adjoint, elliptic, first order differential operator

$$\begin{aligned} D_A\colon\ \Omega^0(M; \mathfrak{g}) \oplus \Omega^1(M; \mathfrak{g}) &\longrightarrow \Omega^0(M; \mathfrak{g}) \oplus \Omega^1(M; \mathfrak{g}) \\ (\alpha, \beta) &\longmapsto (d_A^*\beta, d_A\alpha + *d_A\beta), \end{aligned}$$

where $d_A\colon \Omega^p(M; \mathfrak{g}) \to \Omega^{p+1}(M; \mathfrak{g})$ is the covariant derivative associated to A, and in this case the ρ–invariant is given by

$$\rho_A(M) := \eta(D_A) - \eta(D_\theta),$$

where θ is the trivial connection. While the η–invariant is metric-dependent, the ρ–invariant is not. Since we need topological invariants, we have to use the ρ–invariant rather than the η–invariant. This way we get suitable generalizations of the determinant of the Hessian and the signature of the Hessian at a critical point of the Chern-Simons function.

The group of gauge transformations $\mathcal{G}$ can be identified with $C^\infty(M; \mathfrak{g})$ and acts on $\mathcal{A}$ with infinite-dimensional kernel. By including this gauge equivalence, we can formally rewrite the integral as an integral over $\mathcal{A}/\mathcal{G}$. By further manipulating the expression formally and interpreting the Ray-Singer torsion

$$\tau_M(A) := \prod_k (\det \Delta_A^{(k)})^{(-1)^{k+1}k/2},$$

as a measure on the corresponding connected component $\mathcal{M}(M)_c$ of the gauge equivalence classes of critical points of the Chern-Simons function we get

$$\sum_{c\in C} \frac{1}{|Z(G)|} \int_{A\in\mathcal{M}(M)_c} \sqrt{\tau_M(A)} e^{2\pi i \,\mathrm{CS}_M(A) k} e^{\frac{\pi i}{4}\rho_A(M)} k^{d_c}.$$

Here $\Delta_A^{(k)}$ is the Hodge Laplace operator on $\Omega^k(M;\mathfrak{g})$ coupled to A, $Z(G)$ is the center of G and d_c is the dimension of $\mathcal{M}(M)_c$.

In the case of lens spaces and torus bundles over S^1 [43, 41], the critical points of the Chern-Simons functions are isolated. In [8] we prove this conjecture for finite-order mapping tori and $G = \mathrm{SU}(n)$, which is the first work considering higher-dimensional components. We make use of the fact, that we only need to integrate over irreducible components. For arbitrary G there may be reducible components, otherwise our machinery works for this case as well. We are planning on extending the results from [6, 8] to Seifert fibered manifolds. We are currently working on describing $\mathcal{M}(M)$ in terms of its base and the Seifert invariants.

The SU(2) **Casson invariant and its gauge theoretic successors.** Let M be a homology 3–sphere. In order to make sense of an algebraic count of conjugacy classes of irreducible SU(2) representations of $\pi_1(M)$, we need to consider a Heegaard decomposition $X_1 \cup_\Sigma X_2$ of M and compute the algebraic intersection number of the representation varieties $R^*(X_1)$ and $R^*(X_2)$ of conjugacy classes of irreducible representations inside of $R^*(\Sigma)$ [3, 48]. One of the most remarkable features of the SU(2) Casson invariant is a formula for Dehn surgeries, which makes efficient calculations possible.

Taubes [49] laid the groundwork for new topological invariants motivated by Chern-Simons theory by showing that the SU(2) Casson invariant has a gauge theoretical interpretation as the Euler characteristic of $\mathcal{A}/\mathcal{G}$ in the spirit of the Poincaré-Hopf theorem, where he views the Chern-Simons invariant as an S^1–valued Morse function on $\mathcal{A}/\mathcal{G}$. The critical points of the Chern-Simons function are the flat connections, that is, the connection with curvature 0. The holonomy map is an isomorphism between the flat connections modulo gauge equivalence $\mathcal{M}(M)$ known as the flat moduli space and the representations of the fundamental group modulo conjugacy. Taubes realized that the Hessian of the Chern-Simons invariant and the odd signature operator coupled to the same path of SU(2) connections have the same spectral flow, which is a generalization of the index (relative to a base point). The spectral flow along a path of formally self-adjoint, elliptic differential operators D_t is the algebraic intersection number in $[0, 1] \times \mathbf{R}$ of the track of the spectrum

$$\{(t, \lambda) \mid t \in [0, 1], \lambda \in \mathrm{Spec}(D_t)\}$$

and the line segment from $(0, -\varepsilon)$ to $(1, -\varepsilon)$. Taubes showed that the SU(2) Casson invariant λ can be expressed as

$$\lambda(M) = \frac{1}{2} \sum_{[A]\in\mathcal{M}(M)} (-1)^{\mathrm{SF}(D_{A_t})},$$

where A_t is a path of connections from the trivial connection to A. Floer extended this idea around the same time to instanton Floer homology [26] with a $\mathbf{Z}/8$ grading given by the spectral flow. The Seiberg-Witten revolution in gauge theory shifted attention from instanton Floer homology to the more difficult monopole Floer homology, and progress in either of these versions of Floer homology has been vastly outpaced by the remarkable achievements in the closely related but much more accessible Heegaard-Floer homology introduced by Ozsváth and Szabó. Recently, Kronheimer and Mrowka defined several versions of instanton Floer homology for knots.

Using Taubes's point of view, an SU(3) Casson invariant τ was introduced by [16] and later refined by [18], where suitable correction terms needed to be incorporated to make the invariant independent of perturbations. In the process of understanding this new invariant, a connected sum formula was found [17], and computational tools were developed for Dehn surgeries on $(2, q)$-torus knots [21] as well as for Brieskorn spheres [42, 19, 20]. These papers provide calculation methods for homology spheres obtained by several different cut and paste methods, and the families of examples for which these have yielded calculations show intriguing patterns; they have not, however, led to a conjectural formula for general Dehn surgeries. It is therefore important to continue exploring the behavior of the invariant under further cut and paste constructions.

More recently, Hans Boden and I obtained some results concerning the behavior of τ under the spliced sum construction [22]. Given knots K_1 and K_2 in homology 3-spheres M_1 and M_2, respectively, the spliced sum of M_1 and M_2 along K_1 and K_2 is the homology 3-sphere obtained by gluing the two knot complements along their boundaries matching the meridian of one knot to the longitude of the other. This operation is a generalization of connected sum; indeed when K_1 and K_2 are trivial knots, the spliced sum of M_1 and M_2 along K_1 and K_2 is none other than the connected sum $M_1 \# M_2$.

Casson's invariant, which is additive under connected sum, is also additive under the more general operation of spliced sum by Boyer and Nicas [23] and independently Fukuhara and Maruyama [28]. We computed the SU(3) Casson invariant for spliced sums of a $(2, p)$ and $(2, q)$ torus knot K_1 and K_2 in the 3–sphere as

$$\tau(M) = 16\, \lambda'(K_1)\, \lambda'(K_2), \tag{1}$$

where $\lambda'(K)$ is the SU(2) Casson knot invariant normalized to be 1 for the trefoil. The flat moduli space for $p, q \neq 2$ is more complicated and has a stratified structure. For this reason, the flat moduli space of such a splice sum is necessarily degenerate, and requires perturbation to obtain a finite collection of points to count to evaluate the invariant. Together with Chris Herald we have extended (1) to arbitrary mapping tori.

Since $\mathrm{SL}(2, \mathbf{C})$ is not compact, the flat moduli space is possibly not compact. In fact, every component which is not 0–dimensional is not compact. We can compactify the flat moduli space using the Morgan-Shalen compactification or Lerman's symplectic cutting, but we run into complications, when we perturb. Curtis [25] circumvented this problem by counting only the 0–dimensional components in the same way as the SU(2)–Casson invariant was defined and by ignoring the other components. Together with Andriy Haydys we are in the process of including the non-compact components by an entirely new approach. In all of the gauge theoretic manifestations of the SU(2) Casson invariant and the instanton Floer homology, holonomy perturbations have been used to perturb the flat moduli space. Using a specific Morse function on the moduli space, we are trying to perturb and compactify it in one step without holonomy perturbations.

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