Model structures and their applications

GRK 1150 PhD-seminar

In the past the language of model categories has been an enormously helpful tool to describe phenomena in homotopy theory, and in this seminar we would like to present some examples. The focus of the talks should lie on the applications themselves rather than providing an abstract setup in model category theory. This means that the speaker should emphasize on the question "what can we get out of a certain model structure?" rather than on "How do we verify the axioms in this example?". Each talk is supposed to take about 60 minutes, but definitely **not more** than 90 minutes.

1. **Model categories and chain complexes**

   This talk should introduce the standard model structures on cochain complexes of modules over a ring. At the same time this example should be used to give a revision of the most important definitions in model categories.

   [DS95], chapters 2,3 and 7 [Hov99], 1.1 and 2.3

2. **The homotopy category and derived categories**

   Here we introduce the homotopy category of a model category, again using the example of chain complexes, and further introducing the derived category of ring.

   [DS95], chapters 4 and 5, [Hov99], 1.2 and 2.3, [Kra04] chapter 1

3. **Bousfield localization of spaces**

   Bousfield localization with respect to a generalized homology theory helps to single out phenomena which might not be easy to detect in the global picture. This talk should cover the existence of Bousfield localization and its basic properties together with examples such as p-localization or generally, smashing localizations. It is also the right time to mention the model structure on simplicial sets.

   [Bou75] section 1-3, [Rav84] section 1, [Rav92] chapter 7

4. **Symmetric spectra**

   In previous times it was only known how to construct spectra with a product up to homotopy. Symmetric spectra became one way of describing a category of spectra which carries a strict symmetric monoidal smash product.

   [HSS00]

5. **Ring-, module- and algebra spectra**

   This talk introduces model structures that make it possible to translate concepts from (homological) algebra to stable homotopy theory.

   [SS00]

6. **Diagram categories and homotopy limits**

   Homotopy (co)limits are a tool that often occur in homotopy theory. In this talk we want to describe the setting in which these are defined, using diagram categories, and give explicit examples such as homotopy pushouts and -pullbacks.

   [DS95] chapter 10, [Hov99] 5.1

7. **André-Quillen-cohomology**

   André-Quillen cohomology is a cohomology theory for commutative algebras, replacing the Tor and Ext functors on an abelian category. It can be constructed as cotriple cohomology, using modules of derivations as coefficients. André-Quillen (co)homology groups can be used to obtain information about properties of commutative rings such as regularity or local complete intersection.

   [Qui67] II.§5, [Qui70]

Further topics that could be discussed:

- Quillen's rational homotopy [Qui69]
- Motivic homotopy theory
- Morita theory [SS03]
- Topological Hochschild Homology [Shi00]

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References

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[Kra04] Henning Krause. Derived categories, resolutions and brown representability. *http://www2.math.uni-paderborn.de/people/henning-krause.html*, 2004.

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[Qui69] Daniel Quillen. Rational homotopy theory. *Ann. of Math. (2)*, 90:205–295, 1969.

[Qui70] Daniel Quillen. On the (co-) homology of commutative rings. In *Applications of Categorical Algebra (Proc. Sympos. Pure Math., Vol. XVII, New York, 1968)*, pages 65–87. Amer. Math. Soc., Providence, R.I., 1970.

[Rav84] Douglas C. Ravenel. Localization with respect to certain periodic homology theories. *Amer. J. Math.*, 106(2):351–414, 1984.

[Rav92] Douglas C. Ravenel. *Nilpotence and periodicity in stable homotopy theory*, volume 128 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1992. Appendix C by Jeff Smith.

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