Let K be an ordered field. A function $X \xrightarrow{f} K$ on an arbitrary set K is called *bounded* (or K-bounded if K is not implied by the context) if there is $B \in K$ such that $|f(x)| \leq B$ for all $x \in X$.

Problem 1 (5 points)**.** *For an ordered field K, we equip $\overline{K} = K \cup \{\pm\infty\}$ with the em interval topology: A neighbourhood base of x is*

$$\mathfrak{B}_x = \begin{cases} \left\{(a, \infty]_{\overline{K}} \;\middle|\; a \in K\right\} & x = \infty \\ \left\{[-\infty, a)_{\overline{K}} \;\middle|\; a \in K\right\} & x = -\infty \\ \left\{(a, b]_{\overline{K}} \;\middle|\; a, b \in K \text{ and } a < x < b\right\} & x \in K \end{cases}$$

For $P \in K[T]$ and with the conventions from the lecture, show that $\overline{K} \xrightarrow{P} \overline{K}$ is continuous!

Problem 2 (9 points)**.** *Decide whether the following assertions are true for arbitrary elements $a < b$ of an ordered field K and arbitrary $P \in K[T]$. If true, give a proof. Otherwise give a counterexample.*

- *The function $t \to P(t)$ is bounded on $[a, b]_K$.*
- *If P has no zero in $[a, b]_K$ then the function $t \to 1/P(t)$ is continuous as a map $[a, b]_K \to K$, for the interval topology on source and target.*
- *If P has no zero in $[a, b]_K$ then the function $t \to 1/P(t)$ is bounded on $[a, b]_K$.*

Problem 3 (4 points)**.** *Let R be a ring and $\mathfrak{P}$ a prime cone in R. For $r \in R$, show that the following conditions are equivalent:*

- *$r \in \mathfrak{P}$ and $-r \in \mathfrak{P}$.*
- *$-r^{2n} \in \mathfrak{P}$ for some $n \in \mathbb{N}$.*

Definition 1. *The set of $r \in R$ with these equivalent properties is called the* support *of $\mathfrak{P}$ and denoted* $\operatorname{supp}\mathfrak{P}$.

Problem 4 (3 points)**.** *For a prime cone $\mathfrak{P}$, show that* $\operatorname{supp}\mathfrak{P}$ *is a prime ideal!*

Solutions should be submitted in the exercises on Wednesday, October 25. One of the 21 possible points from this sheet is a bonus point, which does not count in the calculation of the $\geq 50\%$ lower bound of points needed to pass the exercises.