

# Selected Topics in Geometry and Topology (WS 09/10)

## Aspects of metric geometry

Thomas Foertsch

The focus of this lecture lies in exploring curvature- and convexity conditions in the metric space setting.

The two most common non-positive curvature conditions are those due to Alexandrov, on the one hand, and Busemann, on the other hand. While the first one can be described as a uniform convexity of the square of the distance function to points in the space, the latter one is just the convexity of the distance function itself (when restricted to any two affinely parameterized geodesics).

As a four point condition precisely distinguishing between these two non-positive curvature conditions, the so called Ptolemy inequality will also be discussed.

A broad class of metric spaces satisfying this condition, i.e. a broad class of Ptolemy metric spaces, is given by boundaries of  $\text{CAT}(\kappa)$ -spaces,  $\kappa < 0$  (i.e., the boundaries of negatively curved metric spaces), when endowed with their so called Bourdon- and Hamenstädt metrics.

The precise possibilities of structures at infinity of such  $\text{CAT}(\kappa)$ -spaces remain unclear up to now, and the question 'which such structures can be realized at infinity?' is one of the central open questions in this field.

The interplay between the geometry of the boundaries, on the one hand, and the spaces themselves, on the other hand, is another topic covered in the lecturer. Here we will derive certain rigidity theorems.

**In order to follow the lecture, a background in differential geometry is helpful but not necessary!**

The lecture will take place on

**Wednesdays, 10.15-11.45 a.m.**, in the

**Seminar Room S 0.006** in Endenicher Allee 60.

It will start on **Wednesday 21st of October** (i.e. in second week of term!).

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