EXERCISE SHEET NO 4 - ALGEBRAIC TOPOLOGY II

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Exercise 1. Show that the functors colim and lim: $\mathrm{Ab}^I \longrightarrow \mathrm{Ab}$ are right and left exact, respectively, for any shape category I. Show also that colim is exact when $I = \mathbb{Z}$.

Exercise 2. Construct a filtered chain complex (C, F) such that the inclusion does not induce an isomorphism

$$\mathrm{Gr}_{HF} H^*(F_{-\infty}) \to \mathrm{Gr}_{H(C,F)} H^*(C)$$

where we use the (hopefully obvious) notation from the script.

Exercise 3. Fix primes p and q, not excluding $p = q$. Given the chain complex C with $C^0 = C^1 = \mathbb{Z}$, $C^i = 0$ otherwise and differential given by $\mathbb{Z} \xrightarrow{p} \mathbb{Z}$. Filter it by $F_i^0 = 0$ and $F_i^1 = q^i\mathbb{Z}$ for $i \geq 0$ and $F_i^1 = \mathbb{Z}$ for $i \leq 0$. Compute the spectral sequence, the filtrations on the target, and decide whether they are Hausdorff, exhaustive and whether the spectral sequence converges.

Due date: May 22th