

EXERCISE SHEET NO 2 - ALGEBRAIC TOPOLOGY II

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Exercise 1. Let X be a simply connected finite complex. Show that $H^*(\Omega_x X)$ cannot be bounded above for any $x \in X$.

Exercise 2. Let $F \rightarrow E \rightarrow B$ be a fibre sequence of finite complexes with B simply connected. Show that $\chi(E) = \chi(F)\chi(B)$. Bonus question: what happens in the non-simply connected case?

Exercise 3. Compute $H^*(\Omega S^n)$ for all n .

Exercise 4. Compute $H^*(K(\mathbb{Z}, n); \mathbb{Q})$ for all $n \geq 2$ inductively using the Serre spectral sequence of the fibre sequence

$$K(\mathbb{Z}, n) \rightarrow * \rightarrow K(\mathbb{Z}, n+1)$$

from

$$H^*(K(\mathbb{Z}, 1); \mathbb{Q}) = \Lambda_{\mathbb{Q}}[t_1].$$