SCHEDULE FOR THE "WORKSHOP: CATEGORIFICATION"

SCHEDULE LECTURES AND TALKS

Time	Monday	Tuesday	Wednesday
10:00-11:00	**Gukov - part I**	**Ehrig - part II**	**Qi - part II**
11:00-11:30	Coffee/tea	Coffee/tea	Coffee/tea
11:30-12:30	**Ehrig - part I**	**Comes**	**Gukov - part III**
12:30-14:30	Lunch break	Lunch break	Lunch break
14:30-15:30	**Qi - part I**	**Sartori**	**Stroppel - part II**
15:30-16:00	Coffee/tea	Coffee/tea	Coffee/tea
16:00-17:00	**Stroppel - part I**	**Gukov - part II**	**Wedrich**
17:15-18:15	left free	**Queffelec**	left free

Time	Thursday	Friday
10:00-11:00	**Stroppel - Part III**	**Stošić**
11:00-11:30	Coffee/tea	Coffee/tea
11:30-12:30	**Qi - Part III**	**Wilbert**
12:30-14:00	Lunch break	Lunch break
14:00-15:00	**Ehrig - Part III**	**Maksimau**
15:00-15:30	Coffee/tea	Coffee/tea
15:30-16:30	**Carqueville**	**Vaz**
16:30-17:00	left free	left free

ABSTRACTS MAIN LECTURES

Michael Ehrig: Khovanov algebras, category $\mathcal{O}$, and knot invariants.

Abstract: In this series of lectures we will focus on algebraic and Lie theoretic methods in categorification. We will start by reviewing the definition and properties of the (generalized) Khovanov algebra. Furthermore we will discuss the relation between Khovanov algebras and two different categories of representations, category $\mathcal{O}$ for general linear Lie algebra and finite-dimensional representations for general linear Lie super algebras and the two corresponding 2-Kac Moody representations constructed via these categories. If time permits we will outline how this might translate to special orthogonal Lie algebras. Finally we will discuss how Khovanov algebras are related to different generalizations of cobordism categories and corresponding knot invariants.

Sergei Gukov: The geometry of knot homologies and categorification of RTW invariants of 3-manifolds.

Abstract: The physical interpretation of Khovanov-Rozansky homology led to many generalizations and concrete predictions. In particular, it stimulated the formulation of triply-graded HOMFLY-PT homology and colored knot homologies, led to many new differentials (cancelling, universal, colored, exceptional), to new connections with knot contact homology, recursion relations with respect to color-dependence and, most recently, to categorification of RTW invariants of closed 3-manifolds.

Moreover, different ways of looking at this physical setup offer different mathematical formulations of the same homological knot invariants via enumerative geometry, matrix factorizations, vortex counting, etc. They lead to new relations — similar to the relation between Seiberg-Witten and Donaldson-Witten invariants that also represent two faces of the same physical system — and bridges between different areas of mathematics.

In each of my lectures, I will try to focus on one particular aspect, exemplifying concrete predictions and different approaches:

1. Spectral sequences naturally arise when Q-cohomology is "deformed", in particular, when one changes parameters of the system, such as rank, representation, etc. This feature turns out to be a powerful tool that leads to structural properties strong enough to determine many (colored) homologies for knots with up to 10 crossings, even E_6 homology! Whether you were familiar with the subject or knot, I promise to equip you with tools to compute the colored $\mathfrak{sl}(N)$ homology on the back of the envelope and your homework assignment will be to do it for a knot with < 10 crossings. (The main reference for this lecture probably will be http://arxiv.org/pdf/1512.07883.pdf)

2. Matrix factorizations and B-model will lead us to web categories and foam 2-categories studied by Cautis-Kamnitzer-Licata-Morrison, Lauda-Queffelec-Rose, Tubbenhauer-Vaz-Wedrich, and others. The interpretation of these (2-)categories via B-model and matrix factorization suggests natural generalizations, developing which would be a good "homework" project. (The reference for this part is http://arxiv.org/pdf/1507.06318.pdf)

3. There are several ways to approach (and compute!) categorification of RTW invariants of 3-manifolds. One is based on categorification of surgery formulae which, in turn, requires a good control on "color-dependence" of homological knot invariants. This key ingredient is simple and concrete enough and is based on http://arxiv.org/pdf/1203.2182.pdf as well as recent work http://arxiv.org/pdf/1602.05302.pdf. If in Lecture 1 the homework is to compute (colored) homology of a random knot using structural properties, here I will equip you with tools to compute homology categorifying $\mathfrak{sl}(2)$ RTW invariant of a random 3-manifold.

You Qi: Categorification at prime roots of unity.

Abstract: In these three lectures, I'll sketch an algebraic approach to categorify quantum structures at a prime root of unity, which is aimed at eventually lifting the Witten-Reshetikhin-Turaev $3D$

TQFT to a $4D$ TQFT. The talks will be mostly introductory, with examples arising from quantum topology emphasized.

Catharina Stroppel: Approaching the representation theory of Lie superalgebras via categorification.

Abstract: In these talks I will explain a few basic facts about the finite-dimensional representation theory of Lie superalgebras and then describe these categories using categorification methods. In contrast to the classical semisimple case these representations are usually not completely reducible and form an interesting monoidal category. It has connections to classical representation theory like the Brauer and walled Brauer algebras which we will explain. The main focus will then be on the question whether categorification helps to understand the (at least underlying abelian) categories appearing in this context. On the way one obtains a quite unexpected connection to the (classical) geometry of flag varieties, Khovanov algebras and the Kazhdan-Lusztig combinatorics.

ABSTRACTS TALKS

Nils Carqueville: From commutative Frobenius algebras to Gray categories with duals.

Abstract: In this talk I want to explain how topological quantum field theory (TQFT) gives rise to higher categories. According to Atiyah and Segal, a "closed" n-dimensional TQFT is a functor on the n-dimensional bordism category, and it is a standard result that for $n = 2$ closed TQFTs are equivalent to commutative Frobenius algebras. By adding certain additional structure to the 2-dimensional bordism category, one ascends from "closed" to "open/closed" and "defect" TQFTs, and this is paralleled by a lift from algebras to Calabi-Yau categories and pivotal 2-categories. I shall review these constructions in some detail, and then report on recent work which can be viewed as a categorification of the 2-dimensional case: Every 3-dimensional defect TQFT gives rise to a certain 3-category, namely a Gray category with duals.

Jonathan Comes: Oriented Brauer categories and representations of $\mathfrak{gl}(m|n)$ and $\mathfrak{q}(n)$.

Abstract: Given a vector space V, the pure tensor power $V^{\otimes r}$ is equipped with actions by both $\mathfrak{gl}(V)$ and the symmetric group S_r. By classical Schur-Weyl duality, these actions generate the full centralizers of one another. In the 1980s Berele-Regev and Sergeev showed that there is a similar duality between S_r and the Lie superalgebra $\mathfrak{gl}(V)$ when V is a super vector space. Moreover, Sergeev introduced an algebra $Ser(r)$ which plays the role of the symmetric group when $\mathfrak{gl}(V)$ is replaced by the Lie superalgebra $\mathfrak{q}(V)$. In this talk I will explain how the oriented Brauer category plays the role of S_r when we allow for mixed tensor powers $V^{\otimes r} \otimes (V^*)^{\otimes s}$ in the $\mathfrak{gl}(V)$ case. I will then describe a type Q oriented Brauer category which plays the role of $Ser(r)$ in the $\mathfrak{q}(V)$ case.

Ruslan Maksimau: Categorical actions and KLR algebras.

Abstract: The affine Lie algebra $\mathfrak{sl}(n+1)$ contains a subalgebra isomorphic to the affine Lie algebra $\mathfrak{sl}(n)$. It is natural to ask if we can restrict categorical representations from affine $\mathfrak{sl}(n+1)$ to affine $\mathfrak{sl}(n)$. We prove that a category with an action of affine $\mathfrak{sl}(n+1)$ contains a subcategory with an action of affine $\mathfrak{sl}(n)$. To prove this statement, we construct an isomorphism between the KLR algebra associated with the n-cycle and a subquotient of the KLR algebra associated with the $(n+1)$-cycle.

Hoel Queffelec: Braid group metrics from Khovanov-Seidel categorical action.

Abstract: Khovanov and Seidel defined in 2000 a categorical action of the braid group on some category of modules over the zig-zag algebra, a particularly simple quotient of a path algebra. This action, that categorifies the Burau representation, turns out to be faithful. We are interested in the metrics on the braid group that can be deduced from this representation. Indeed, the zig-zag algebra can be endowed with several gradings, together with its representation category, inducing the notion of spread of a complex under the action of a braid. I'll explain how one can recover the word-length in the usual and dual generators from two different gradings, and, time permitting, illustrate how this can be helpful for classical questions. (This is joint work with A. Licata.)

Antonio Sartori: Link invariants of type A and categorification.

Abstract: We describe the finite-dimensional representation category of $\mathfrak{gl}(m|n)$ and of its quantized enveloping algebra using variations of Howe duality, and we review the Reshetikhin-Turaev construction of the corresponding link invariants of type A. We discuss then some results (and some open questions) on their categorification, in particular using the BGG category $\mathcal{O}$.

Marko Stošić: Thick calculus, colored HOMFLY-PT homologies, and beyond.

Abstract: In the first part of the talk, I will present some basic results concerning the so-called thick (or extended) diagrammatical calculus of categorified quantum $\mathfrak{sl}_2$ and $\mathfrak{sl}_n$. Apart from the direct categorification of quantum groups, this calculus has its applications in the definition of the categorification of $\mathfrak{sl}_n$ link polynomials colored colored by fundamental colors (specializations of colored HOMFLY-PT polynomials). In the second part of the talk, some of the properties of the colored HOMFLY-PT homologies will be presented based on the conjectural relationship with the string theory and BPS homology. This implies the existence of rich and rigid structural properties of such homologies. In turn, this also gives explicit expressions for the colored HOMFLY-PT polynomials for large classes of knots, as well as explicit computation of BPS numbers together with many surprising integrality properties.

Pedro Vaz: Categorification of Verma modules.

Abstract: In this talk I will start by explaining the categorification of the n-dimensional irreducible representations of quantum $\mathfrak{sl}_2$ using cohomologies of finite-dimensional Grassmannians and partial flag varieties, due to Chuang-Rouquier and Frenkel-Khovanov-Stroppel. I will then present a generalization of their construction to categorify (all) the Verma modules for quantum $\mathfrak{sl}_2$ and explain how to extend it to other quantum Kac-Moody algebras.

Paul Wedrich: Some differentials on colored Khovanov-Rozansky homology.

Abstract: Colored Khovanov-Rozansky homologies categorify the Reshetikhin-Turaev $\mathfrak{sl}(N)$ invariants of links with components colored by exterior power representations of $\mathfrak{sl}(N)$. Colored HOMFLY homologies categorify the large N limits of the latter. In this talk, I will introduce some new spectral sequences between members of this family of link homologies, building on earlier work of Jake Rasmussen and on joint work with David E. V. Rose. In particular, these spectral sequences allow a proof of a conjecture of Gorsky, Gukov and Stošić about the exponential growth of colored HOMFLY homologies of knots.

Arik Wilbert: Two-block Springer fibers and Springer representations in type D.

Abstract: We explain how to construct an explicit topological model (similar to the topological Springer fibers of type A appearing in work of Khovanov and Russell) for every two-block Springer fiber of type D (as well as type C) and sketch how one proves that the respective topological model is indeed homeomorphic to its corresponding Springer fiber. As an application it is discussed how these combinatorially defined topological Springer fibers can be used to reconstruct the famous Springer representation in an elementary and explicit way.