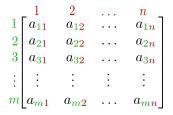
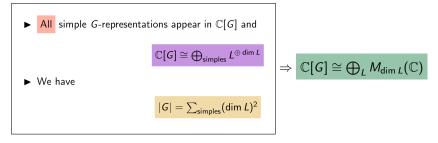
What is...a block?

Or: Decomposing a problem

Matrices? Well...

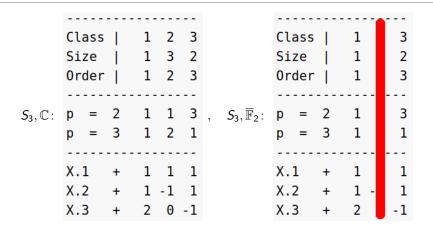


 \blacktriangleright Over $\mathbb C$ we get a decomposition into matrices



▶ In general $\mathbb{K}[G] \cong \bigoplus_{i=1}^{k} B_i$ with B_i indecomposable Blocks

Let us look at S_3



▶ $\mathbb{C}[S_3] \cong M_1(\mathbb{C}) \oplus M_1(\mathbb{C}) \oplus M_2(\mathbb{C}) \Rightarrow$ blocks are matrix algebras

▶ $\overline{\mathbb{F}}_2[S_3] \cong \overline{\mathbb{F}}_2[X]/(X^2) \oplus M_2(\overline{\mathbb{F}}_2) \Rightarrow$ blocks are not necessarily matrix algebras (The projective cover of the trivial rep is of dim 2)

S₃ continued

	Class	1 2 3	Class	12
	Size	1 3 2	Size	1 3
	Order	1 2 3	Order	12
S_3, \mathbb{C} :	p = 2	1 1 3	$, S_3, \overline{\mathbb{F}}_3: p = 2$	1 1
	p = 3	1 2 1	p = 3	12
				·
	X.1 +	1 1 1	X.1 +	1 1
	X.2 +	1 -1 1	X.2 +	1 -1
	X.3 +	2 0 -1	X.3 +	20-

 $\blacktriangleright \ \mathbb{C}[S_3] \cong M_1(\mathbb{C}) \oplus M_1(\mathbb{C}) \oplus M_2(\mathbb{C}) \Rightarrow \text{blocks are matrix algebras}$

► $\overline{\mathbb{F}}_3[S_3] \cong (\overline{\mathbb{F}}_3[X]/(X^3) \oplus \overline{\mathbb{F}}_3[X]/(X^3)) \Rightarrow$ no matrix blocks $(\chi_3 = \chi_1 + \chi_2 \mod 3)$ For \mathbb{K} algebraically closed of characteristic *p*:

► There is an up to reordering unique decomposition

 $\mathbb{K}[G] \cong \bigoplus_{i=1}^{\kappa} B_i$ B_i is an indecomposable two-sided ideal summand

Existence and uniqueness

- ▶ Primitive central idempotents $\stackrel{1:1}{\longleftrightarrow}$ blocks Construction
- ► The simples are partitioned by the blocks Decomposition of the problem
- \blacktriangleright Simple characters/ $\mathbb C$ fit into blocks via simple Brauer characters

T := CharacterTable(SymmetricGroup(10)); Blocks(T,3);

```
[
{
1, 2, 5, 6, 9, 10, 11, 12, 13, 14, 19, 20, 21, 22, 29, 30, 33, 34, 35, 38,

39, 42 },

{ 3, 8, 15, 18, 23, 26, 27, 31, 37 },

{ 4, 7, 16, 17, 24, 25, 28, 32, 36 },

{ 40 },

{ 41 }
```

- The principal block $B_{triv} =$ block of the trivial rep
 - B_{triv} is the "most complicated" block
- $B_{triv} \cong \mathbb{K} \Leftrightarrow \mathbb{K}[G]$ is semisimple

Thank you for your attention!

I hope that was of some help.