What are...Green cells?

Or: Measuring information loss

Information loss after multiplication

- Multiplication in a monoid often destroys information
- Cells can then be thought of as keeping track of the information loss
- Cells order the monoid into equivalence classes of equal information

Left, right, two-sided cells

$$x \leq_L y \Leftrightarrow \exists z : y = zx$$

$$x \leq_R y \Leftrightarrow \exists z' : y = xz'$$

$$x \leq_{LR} y \Leftrightarrow \exists z, z' : y = zxz'$$

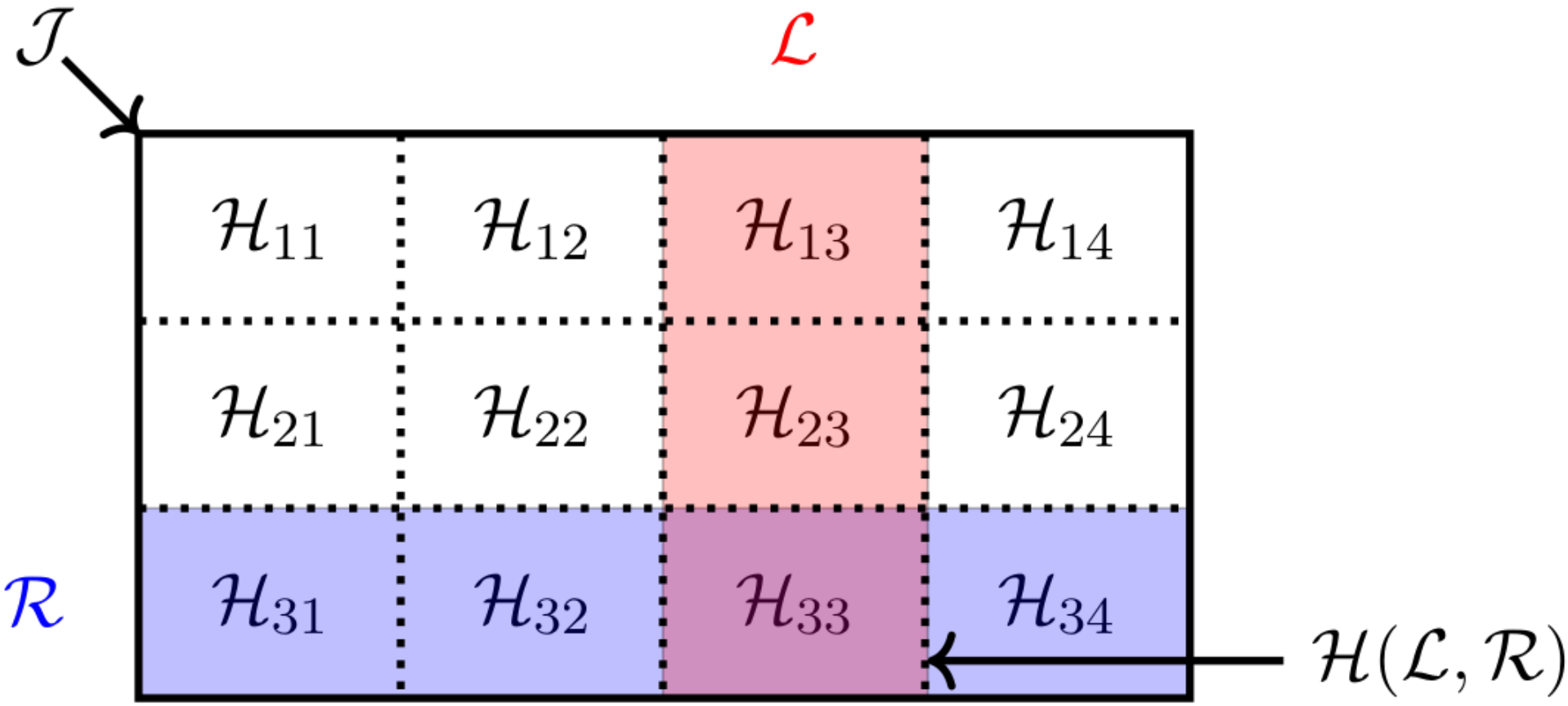
$$x \sim_L y \Leftrightarrow (x \leq_L y) \wedge (y \leq_L x)$$

$$x \sim_R y \Leftrightarrow (x \leq_R y) \wedge (y \leq_R x)$$

$$x \sim_{LR} y \Leftrightarrow (x \leq_{LR} y) \wedge (y \leq_{LR} x)$$

- If $y = zx$, then y can be obtained from x by left multiplication, and we can say that y is left bigger than x
- If one can go back we say $x \sim_L y$ Left cells $\mathcal{L}$
- In a group we can go back by $z^{-1}y = x$ so x is always left equivalent y
- Similarly for right $\mathcal{R}$ and two-sided $\mathcal{J}$

Matrices in cells



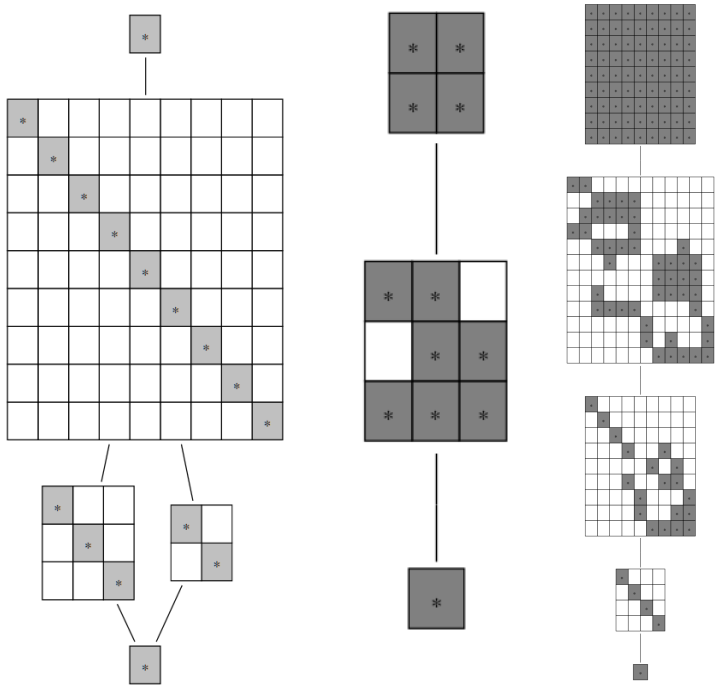
- H-cell = an intersection of a left and a right cell
- Let's repeat Cells order the monoid into equivalence classes of equal information
- The classes give a matrix-type decomposition of the monoid

For completeness: A (primer of a) formal statement

Green cells = Green's relations satisfy:

- Every $H/L/R$-cell is contained in some J-cell
- Every J-cell is a disjoint union of $H/L/R$-cells
- Every H-cell contains either one or no idempotent $e^2 = e$
- If $\mathcal{H}(e)$ contains an idempotent, then $\mathcal{H}(e)$ is a maximal subgroup

Color idempotent H-cells:



Cyclic monoids

$$\mathcal{J}_t \quad a^3, a^4 \quad \mathcal{H}(e) \cong \mathbb{Z}/2\mathbb{Z}$$

$$\mathcal{J}_{a^2} \quad a^2$$

$$\mathcal{J}_a \quad a$$

$$\mathcal{J}_b \quad 1 \quad \mathcal{H}(e) \cong 1$$

- Cyclic monoids $C_{i,k} = \langle a | a^{i+k} = a^i \rangle$ of cardinality $i + k$
- $C_{3,2} = \{1, a, a^2, a^3, a^4\}$
- The cell structure is always as above with H-groups 1 and $\mathbb{Z}/k\mathbb{Z}$

Thank you for your attention!

I hope that was of some help.