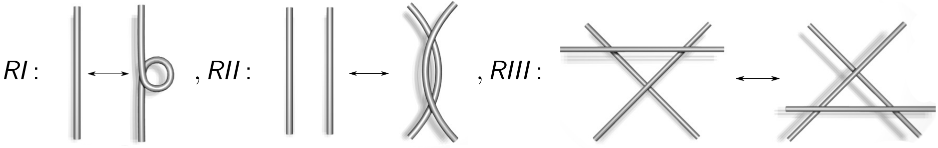
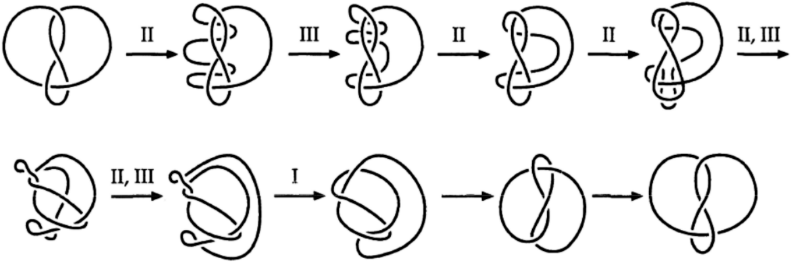
What is...$(236c)^{11}$?

Or: That is a polynomial in c!

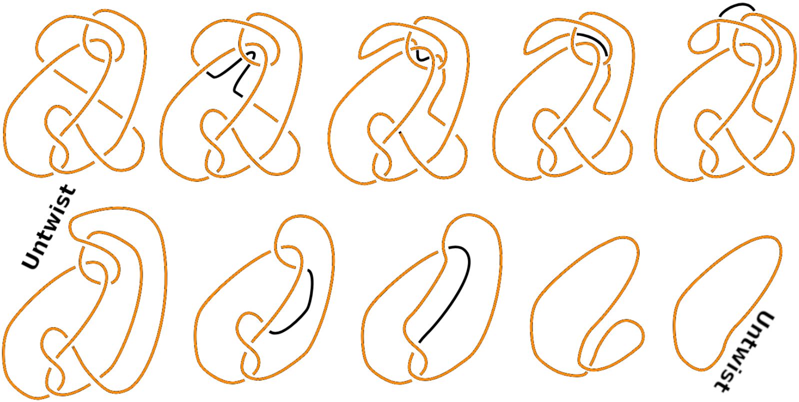
Reidemeister moves





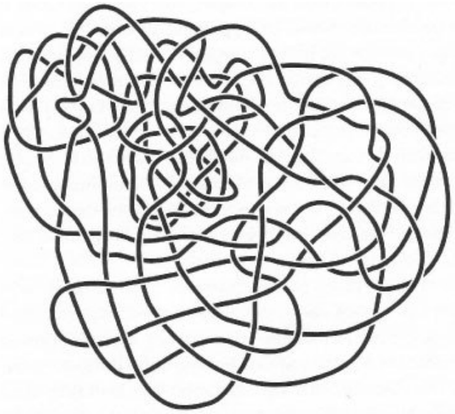
- Reidemeister theorem Two shadows present the same knot ⇔ they are related by RI, RII, RIII moves and isotopies
- Question How many Reidemeister moves do we need to go between shadows?

Shadows of the unknot



- The above shadow of the unknot is called a culprit
- We need 10 Reidemeister moves to undo it
- Question How many Reidemeister moves do we need at most to undo all shadows of the unknot?

Well, that seems to be hard...



- The above shadow of the unknot needs a lot of Reidemeister moves to undo it
- Is there any hope to get a bound in the number of crossings?

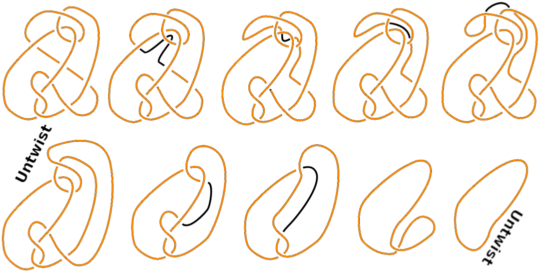
Enter, the theorem

Any shadow of the unknot with c crossings can be undone using

at most $(236c)^{11}$ Reidemeister moves

- The upper bound often overcounts:

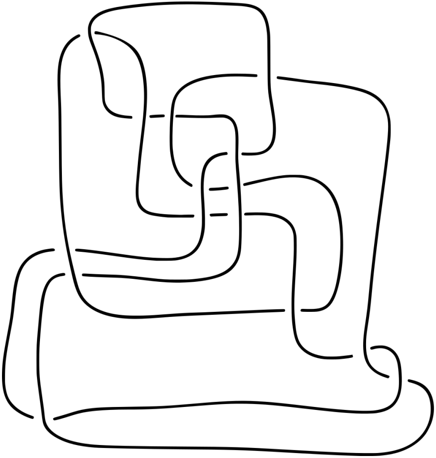
Need 10 Reidemeister moves



The bound is 1264829635273177124160143360000000000 Reidemeister moves

- We never need more than $(7c)^2$ crossings for the shadows during undoing
- Great upshot: testing unknottedness can be done in exponential time

Unknotting is hard...



- There are "hard" unknot shadows for any big enough c
- One needs at least $\frac{1}{25}c^2$ moves as an upper bound
- Unknotting is in co-NP (proven ∼2021), but not much more is known

Thank you for your attention!

I hope that was of some help.