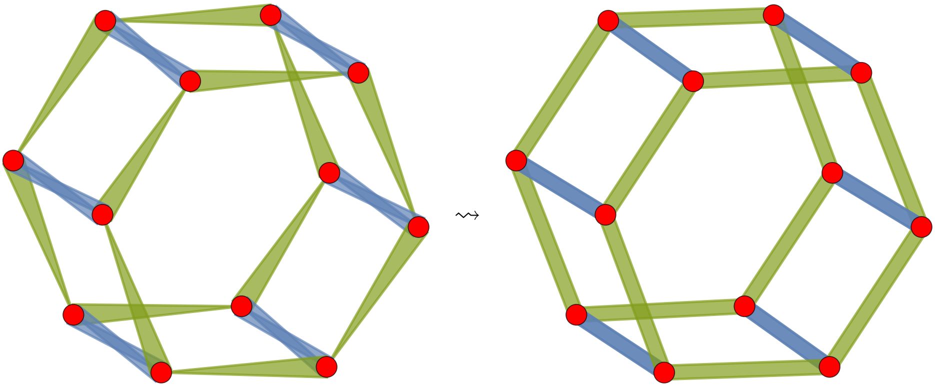
What is...group growth?

Or: Small, medium-sized and big groups?

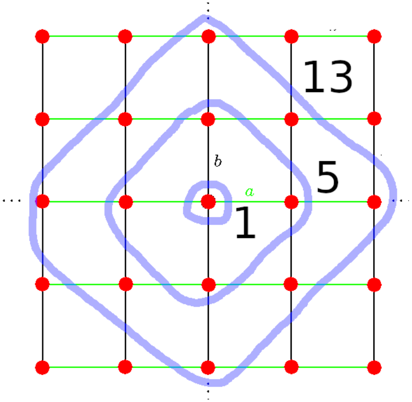
Cayley graphs again



$D_6 = \langle a, b | a^6 = b^2 = (ab)^2 = 1 \rangle$ $\rightsquigarrow$ $D_6 = \langle a, a^{-1}, b \rangle$

- For a group consider the Cayley graph
- We only want Cayley graphs for symmetric generating sets
- This just means we consider Cayley graphs as unoriented

Distance from the origin



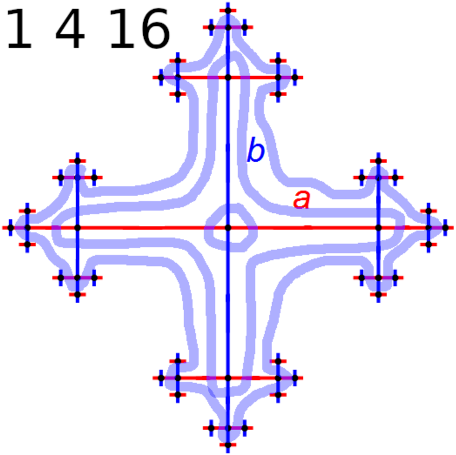
growth of $\mathbb{Z}^2 \approx n^2$

- Growth of a group (for a fixed Cayley graph Γ) = growth of

  $f(n)$ = number of vertices of distance n from the origin in Γ

- Example Finite groups have constant growth
- Example $\mathbb{Z}^k$ has polynomial growth

Exponential growth



growth of $F_2 \approx 2^n$

- Example F_k for $k > 1$ has exponential growth
- Question What growth rates occur? Constant, polynomial, exponential...
- Question What about intermediate growth?

Enter, the theorem(s)

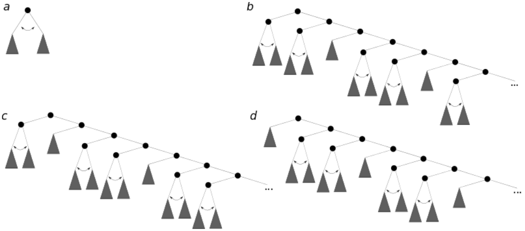
(i) The whole setting is independent of the choice of Cayley graph

(ii) Constant growth happens if and only if the group is finite

(iii) Polynomial growth happens if and only if the group is virtually nilpotent

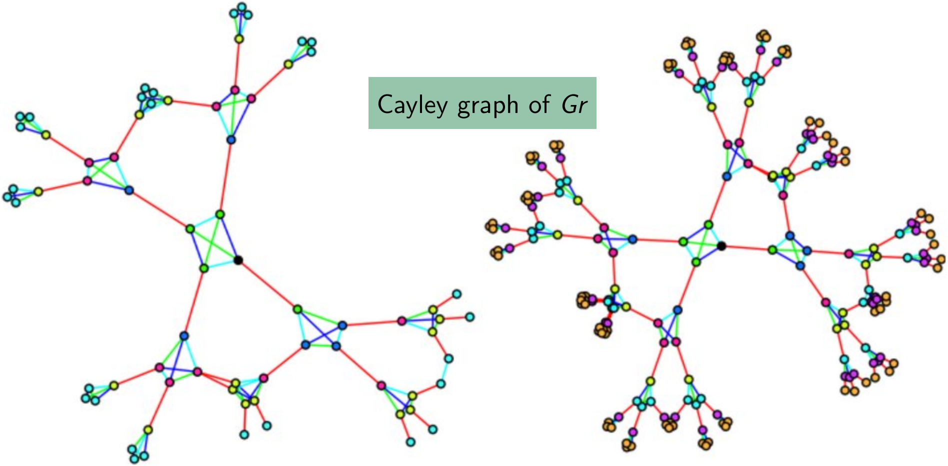
(iv) Every finitely generated group has at most exponential growth

(v) Question ∼1968, answer ∼1984: there exist groups of intermediate growth



- A complete picture of which orders of growth are possible is missing
- Conjectural, there are no finitely presented groups of intermediate growth

Grigorchuk group *Gr*



- Grigorchuk group = certain subgroup of automorphisms of a binary tree
- Theorem *Gr* has intermediate growth $\exp(n^{0.504}) \leq \text{growth} \leq \exp(n^{0.768})$

Thank you for your attention!

I hope that was of some help.