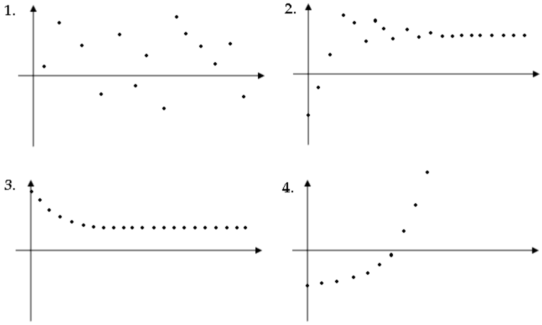
What is...a limit line graph?

Or: Line graphs converge; in some sense...

Limits of sequences



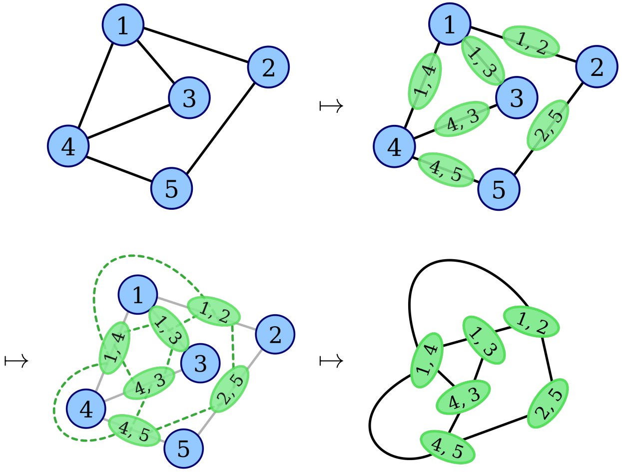
- In a letter written to A. Holmboe on January 16, 1826, Abel declared that

  Divergent series are in general deadly,

  and it is shameful that anyone dare to base any proof on them

- Fabulous, so let us investigate sequences (not series!) in graph theory
- Spoiler Graphs are better behaved than numbers, *e.g.* pattern 1. does not

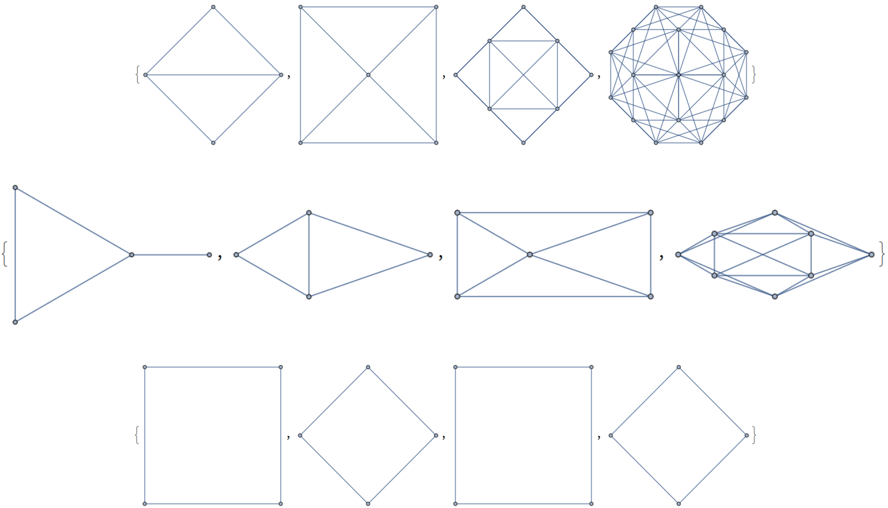
Line graphs



The line graph $L(G)$ of a graph G has:

- Vertices being the edges of G
- Edges depending on whether the edges of G share a vertex

A sequence of line graphs

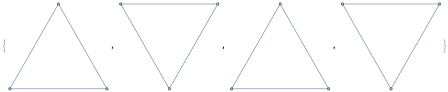


- Consider the sequence $L^k(G) = L(L(...L(G)))$
- Goal Study $L^\infty(G) = \lim_{k\to\infty} L^k(G)$

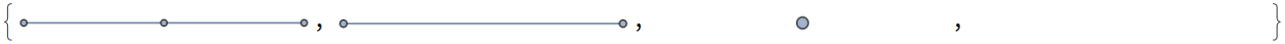
Enter, the theorem

For finite connected graphs $L^{\infty}(G)$ shows only four patterns:

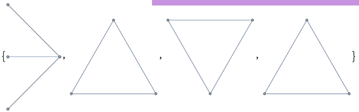
(i) For cycles the sequence is constant



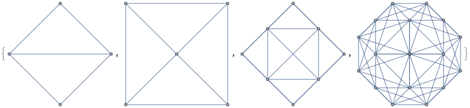
(ii) For lines the sequence's limit is empty



(iii) For the claw/D_4 the sequence is effectively constant

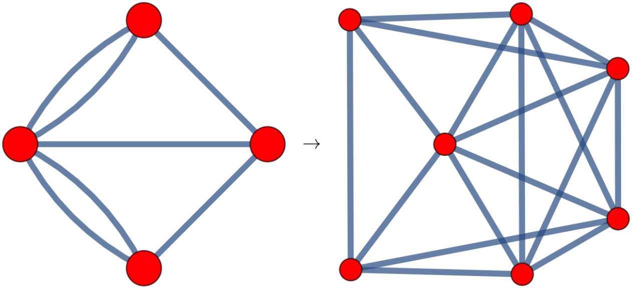


(iv) Otherwise the sequence's limit is infinite



The limit is Hamiltonian

Wait, aren't these dual problems?



$G \to L(G)$

- These problems are not dual in any known way
- G Eulerian $\Rightarrow$ its line graph $L(G)$ is Hamiltonian
- $L(G)$ Hamiltonian $\not\Rightarrow$ G is Eulerian

- Hamiltonian = has a path that visits each vertex exactly once
- Theorem For all finite connected graphs, $G \neq$ line, $L^{\gg 1}(G)$ is Hamiltonian

Thank you for your attention!

I hope that was of some help.