What is...the Robinson-Schensted correspondence?

Or: Boxes and permutations

## Young diagrams (YD)



- Young diagram = boxes arranged in left-justified nonincreasing rows
- Young diagrams are everywhere in combinatorics
- Careful There are three conventions: English, French and Russian


## Young tableaux (YT)

| 1 | 4 | 5 | 7 | 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 6 | $\sqrt{ }$ |  |  |  |
| 3 |  |  |  |  |  |
| 9 |  |  |  |  |  |


| 1 | 4 | 7 | 5 | 8 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 6 |  |  |  |  |
| 3 |  |  |  |  |  |
| 9 |  |  |  |  |  |


| 1 | 4 | 5 | 5 | 8 |
| :--- | :--- | :--- | :--- | :--- |
| 2 | 6 |  |  |  |
| 3 |  |  |  |  |
| 9 |  |  |  |  |

- Tableaux $=$ fill boxes with numbers $\{1, \ldots, n\}$
- Standard tableaux $=$ non-repeating, numbers in rows and columns increase


## A funny count



- $\left|S_{n}\right|=n!=\sum_{\mathrm{YD} \text { of } n}|\mathrm{YT}|^{2} \Rightarrow$ pairs of YT count permutations
- Task Find an explicit bijection


## Enter, the theorem

## There is an explicit bijection

## Permutations $\rightarrow$ pairs of $\mathrm{YT}(P, Q)$

with an explicit inverse
Permutations $\leftarrow$ pairs of $\mathrm{YT}(P, Q)$

- The algorithm is best explained via example (next slide)

- If $\sigma \mapsto(P, Q)$, then $\sigma^{-1} \mapsto(Q, P)$
- There are many other important properties


## Schensted and Viennot

| 1 | 2 | 5 | 7 |
| :--- | :--- | :--- | :--- |
| 3 | 8 |  |  |
|  |  |  |  |
|  |  |  |  |

(4)

S:

| 1 | 2 | 4 | 7 |
| :--- | :--- | :--- | :--- |
| 3 | 5 |  |  |
|  |  |  |  |

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- Schensted's algorithm (S) bumps and records
- Viennot's algorithm (V) uses a grid

Thank you for your attention!

I hope that was of some help.

