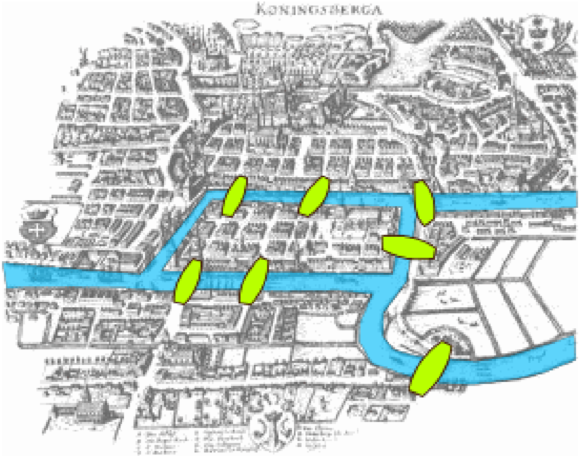
What is...the BEST theorem?

Or: de [B]ruijn, van Aardenne-[E]hrenfest, [S]mith, [T]utte

Euler and Königsberg



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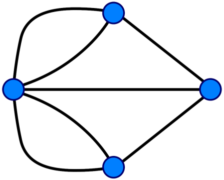
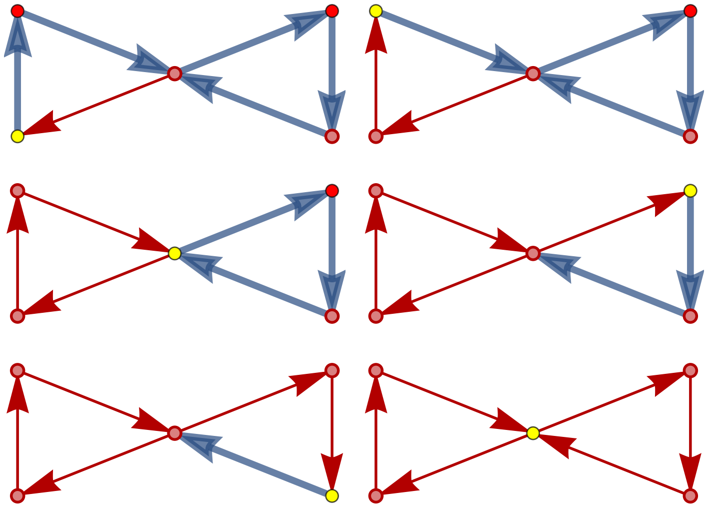


There is no such cycle

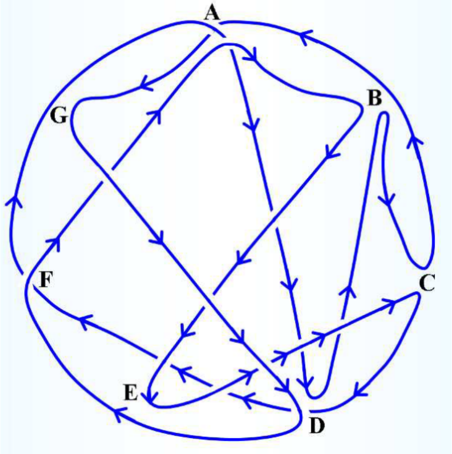
- An Eulerian cycle in a graph visits every edge exactly once
- There is an easy criterion to decide whether a graph is Eulerian
- Task Count all Eulerian cycles

The directed version



- The directed version turns out to be easier
- **Task (as before)** Count all Eulerian cycles

Counting made easy?



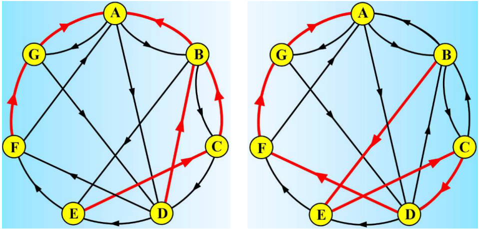
- The above graph has 16 Euler cycles
- We can know that without counting them!

Enter, the (BEST) theorem

The number $ec(G)$ of Eulerian cycles in a connected Eulerian graph G is

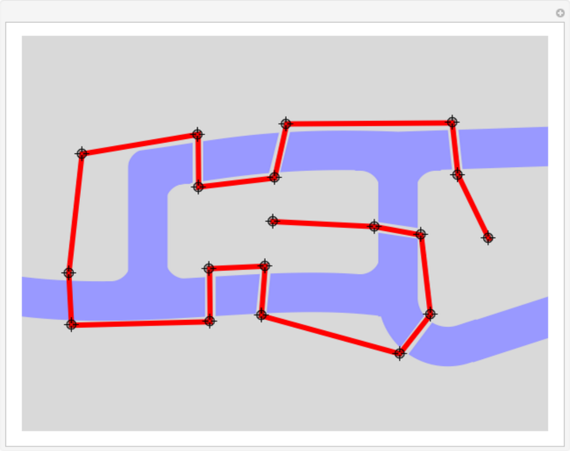
$$ec(G) = t_A \prod_{v \in G} (\deg v - 1)!$$

- t_A = number of spanning trees directed toward A Easy to compute



- $\deg v$ = degree of a vertex (out=in for a Eulerian graph) Easy to compute
- In our example, $t_A = 2$, $\deg A = \deg B = \deg D = 3$, $\deg C = \deg E = \deg F = \deg G = 2$, so we get $ec(G) = 16$

Directed versus undirected



- Counting the number of directed Eulerian cycles is easy by BEST
- Counting the number of undirected Eulerian cycles is hard #P complete

Thank you for your attention!

I hope that was of some help.