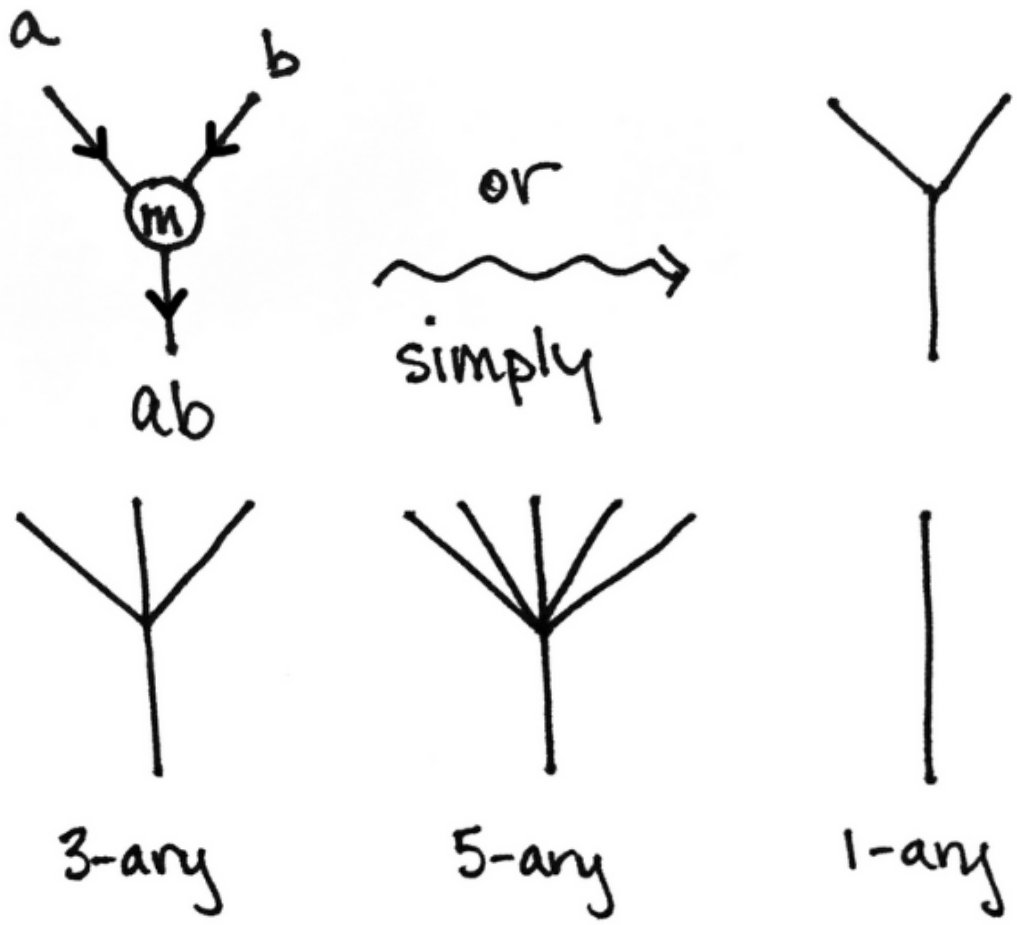
What are...operads?

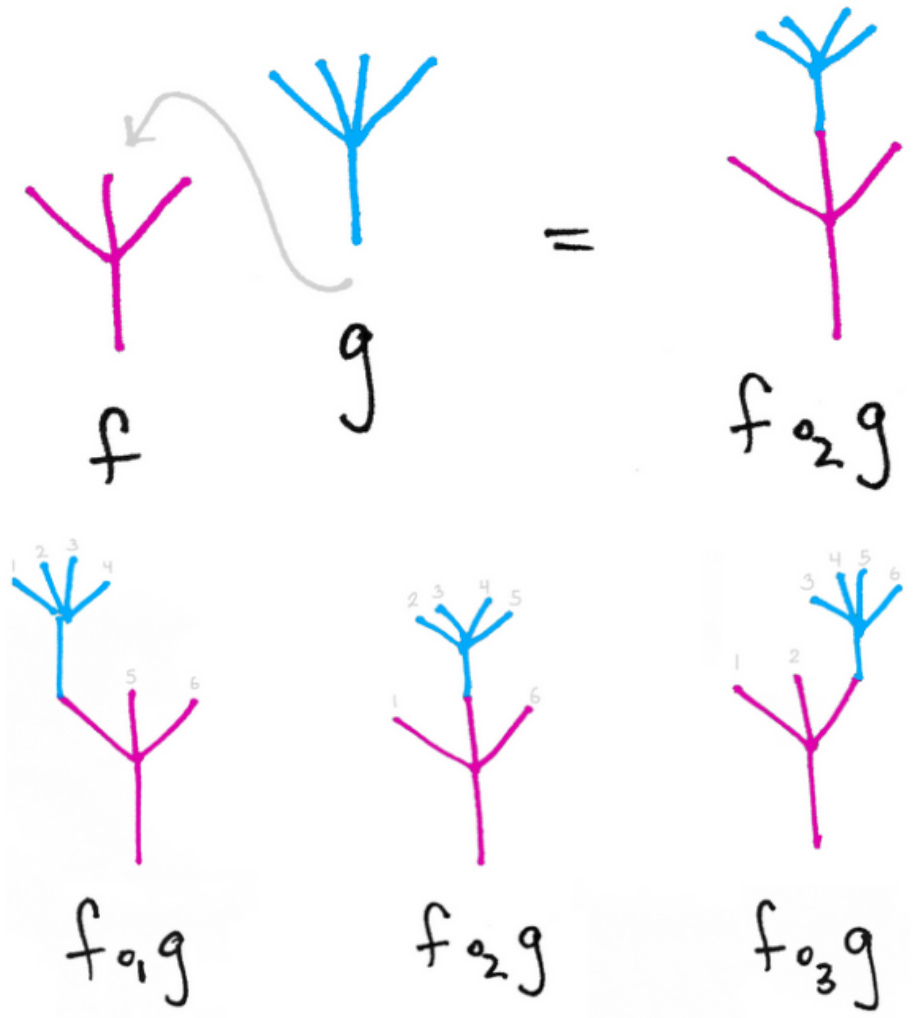
Or: Topological algebra

Trees and algebra



- We can think of multiplication as a tree
- More (or fewer) than 2 inputs can also be handled by trees
- Task Describe the algebra of trees

Composition? Sure thing!



- Label the inputs of trees by numbers
- Stacking gives us many composition operations

More than trees? Sure thing!

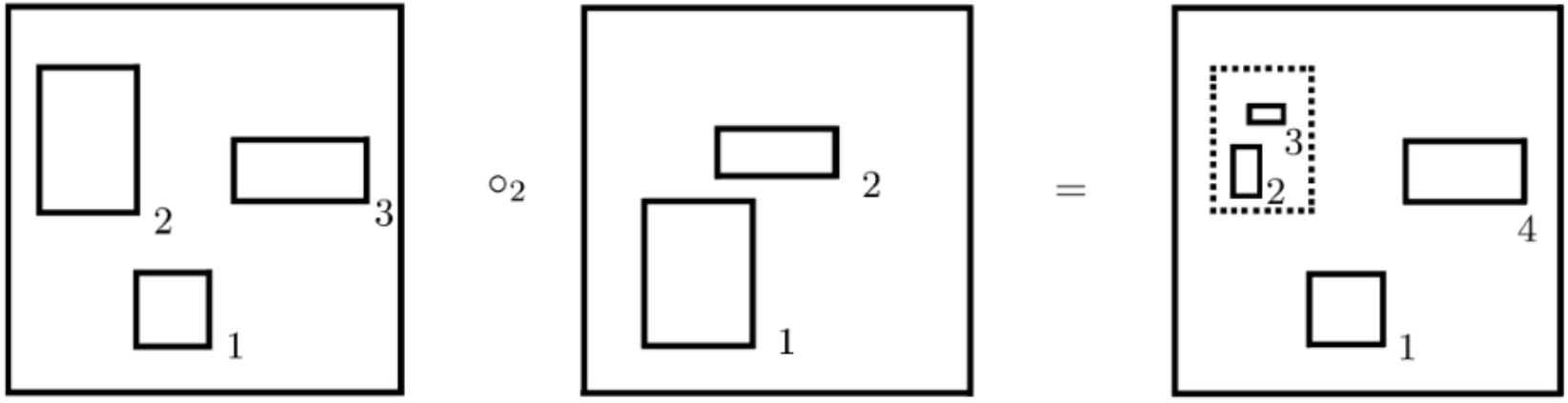
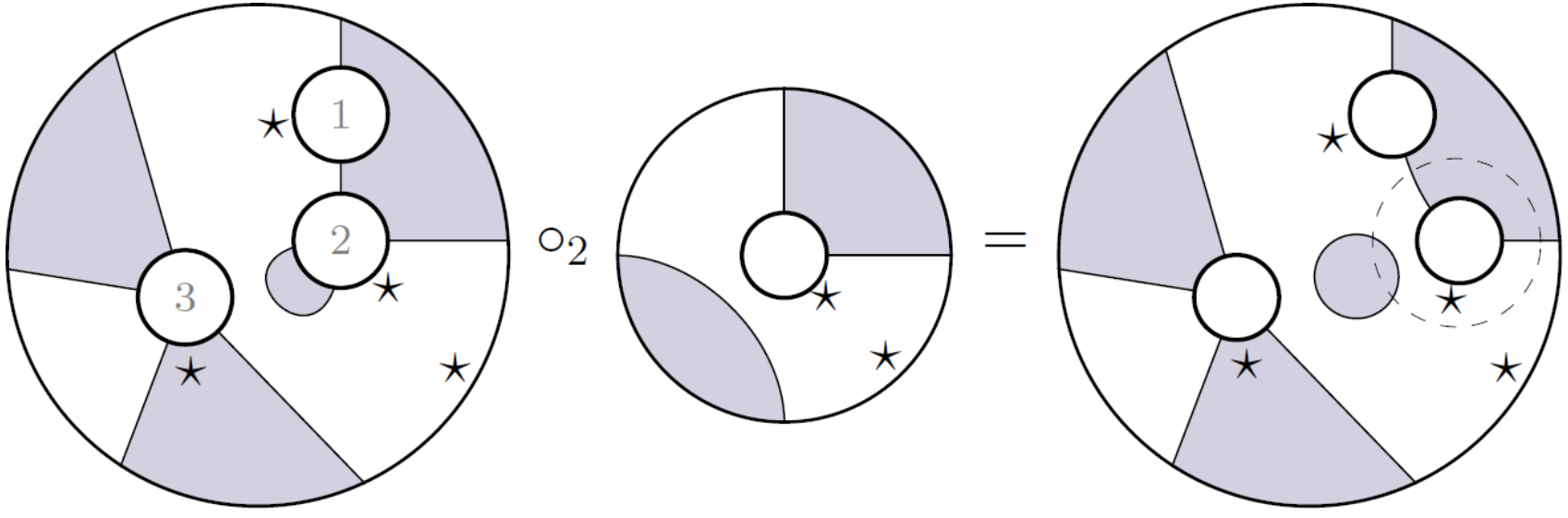


Figure 2. The little 2-cubes operad.



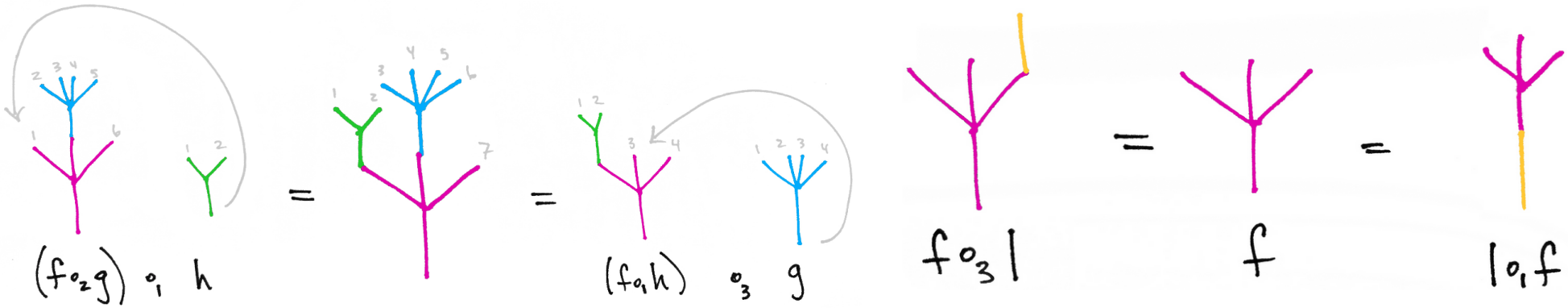
- There are many examples fitting in the same definition
- Basically, everything with some "scaling similarity" works

Enter, the theorem

A (nonsymmetric) operad is a sequence of sets $\left(P(i)\right)_{i\in\mathbb{N}}$ with a unit $1 \in P(1)$ and operations

$$P(n) \times P(k_1) \times \ldots \times P(k_n) \to P(k_1 + \ldots + k_n)$$

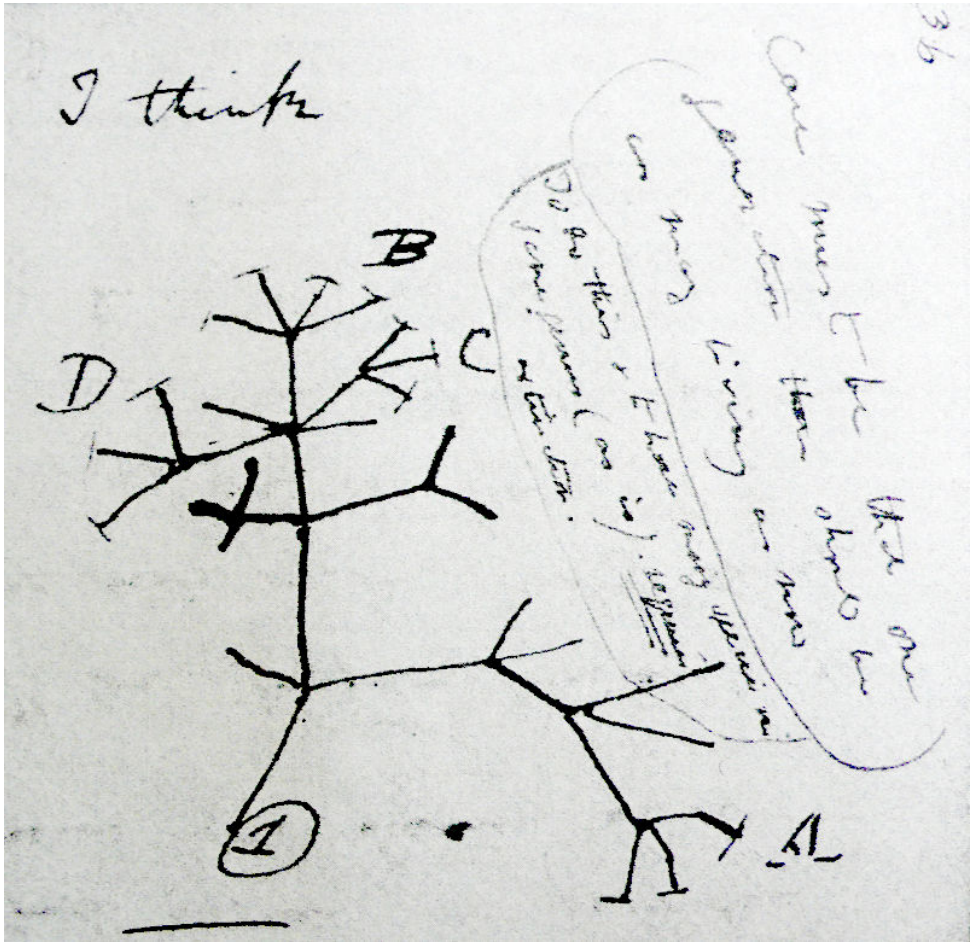
such that associativity and unitality hold, which basically means:

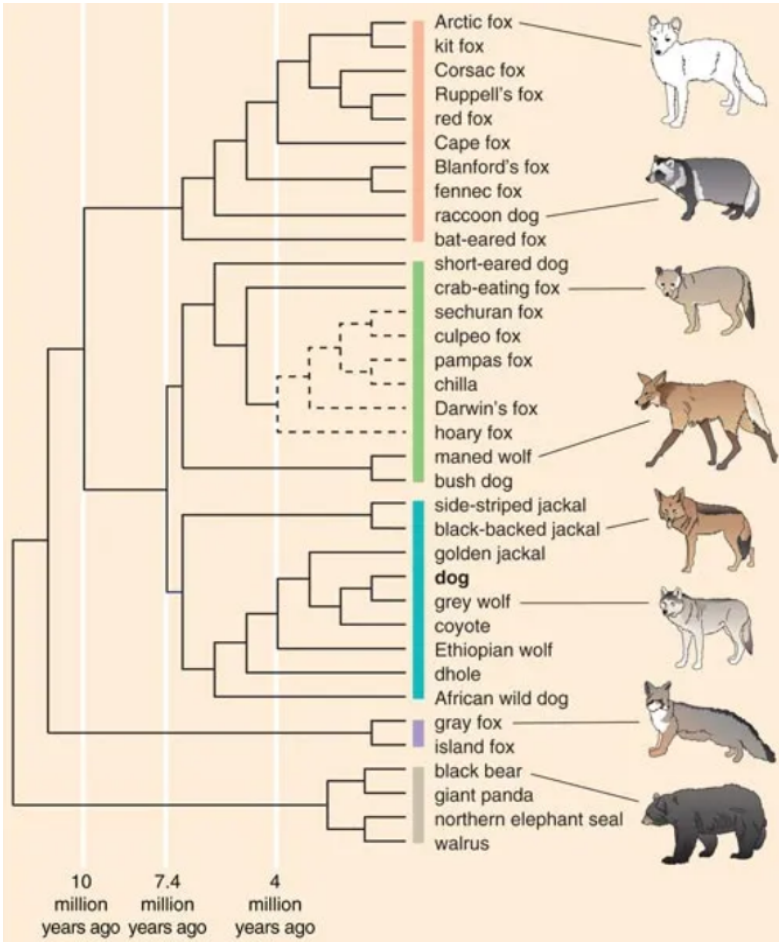


"These are useful Theorem", *e.g.*

Operads are particularly important and useful in categories with a good notion of homotopy, where they play a key role in organizing hierarchies of higher homotopies (Stasheff 2004)

The phylogenetic tree





- Operads are used to model many different things

"These are useful Theorem"

- For example, the phylogenetic tree can be modeled via operads

Thank you for your attention!

I hope that was of some help.