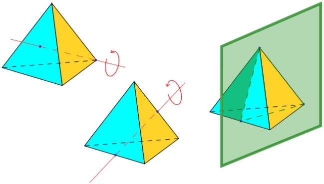
What is...the gnu function?

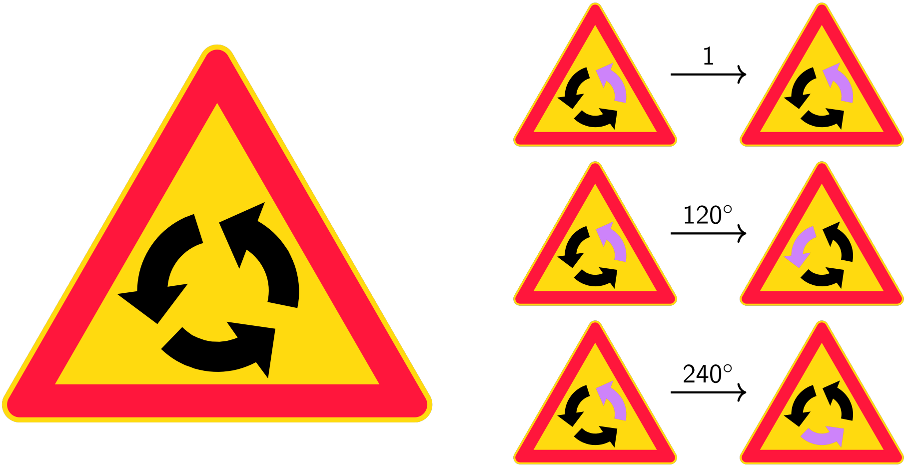
Or: Symmetries are wild

Symmetries

	Abstract	Incarnation
Numbers	3	 or...
Groups	$S_4 = \langle s, t, u \mid \text{some relations} \rangle$	 or...

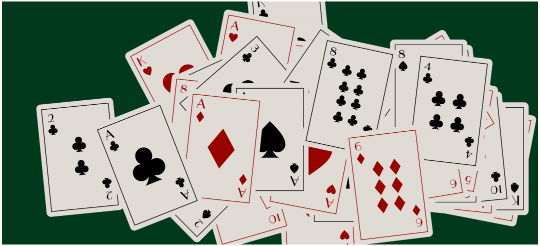
- Groups formalize the concept of symmetry
- Goal Count the number of different symmetries of order n
- $gnu(n) \rightsquigarrow$ number of symmetries of order n

Rotational symmetries



- Rotational symmetries imply $gnu(n) \geq 1$
- A bit of abstract reasoning shows $gnu(prime) = 1$
- Calculating other values of $gnu(n)$ is hard

Symmetries are everywhere



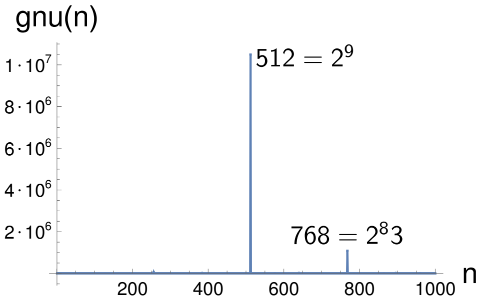
- Symmetries are everywhere in mathematics and real life
- Computing $gnu(n)$ should be (and is!) very hard
- Surprise One can prove nontrivial facts about $gnu(n)$

Enter, the theorem

For p a prime we have:

Asymptotically $gnu(p^k) \approx p^{2k^3/27+O(k^{8/3})}$

- This is a striking pattern in a sea of randomness:



1 1	1 2	1 3	2 4	1 5	2 6	1 7	5 8	2 9	2 10	1 11	5 12
1 13	2 14	1 15	14 16	1 17	5 18	1 19	5 20	2 21	2 22	1 23	15 24
2 25	2 26	5 27	4 28	1 29	4 30	1 31	51 32	1 33	2 34	1 35	14 36
1 37	2 38	2 39	14 40	1 41	6 42	1 43	4 44	2 45	2 46	1 47	52 48
2 49	5 50	1 51	5 52	1 53	15 54	2 55	13 56	2 57	2 58	1 59	13 60
1 61	2 62	4 63	267 64	1 65	4 66	1 67	5 68	1 69	4 70	1 71	50 72
1 73	2 74	3 75	4 76	1 77	6 78	1 79	52 80	15 81	2 82	1 83	15 84
1 85	2 86	1 87	12 88	1 89	10 90	1 91	4 92	2 93	2 94	1 95	231 96
1 97	5 98	2 99	16 100	1 101	4 102	1 103	14 104	2 105	2 106	1 107	45 108
1 109	6 110	2 111	43 112	1 113	6 114	1 115	5 116	4 117	2 118	1 119	47 120
2 121	2 122	1 123	4 124	5 125	16 126	1 127	2328 128	2 129	4 130	1 131	10 132
1 133	2 134	5 135	15 136	1 137	4 138	1 139	11 140	1 141	2 142	1 143	197 144

- The 11758615 groups of order <1000 are swamped by the 10494213 of order 512

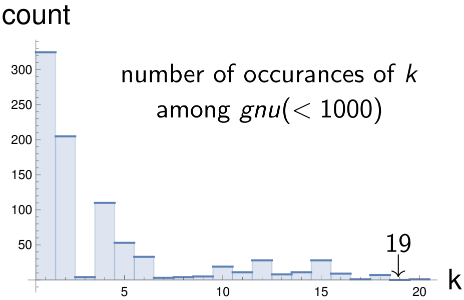
Hunting gnus

We now formulate the *gnu-hunting conjecture.*

Conjecture 8.1. *Every positive integer is a group number.*

21.6 Surjectivity of the enumeration function

An entertaining but mildly eccentric question which has been raised by a number of people at different times (for example, there are faint intimations



- The above conjecture has been verified for numbers up to 10000000
- Proving it will probably take a while...

Thank you for your attention!

I hope that was of some help.