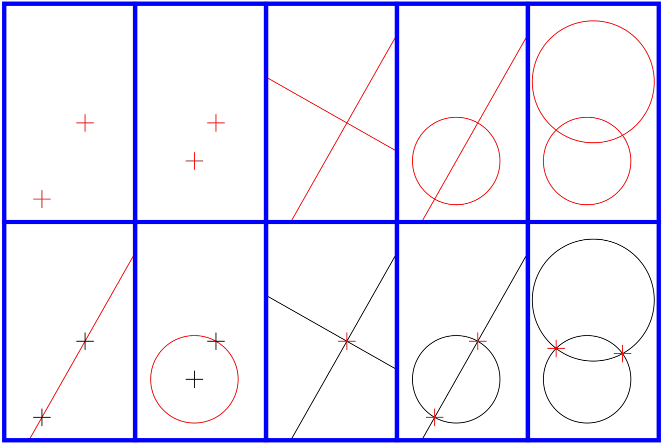
# What is...the Gauss–Wantzel theorem?

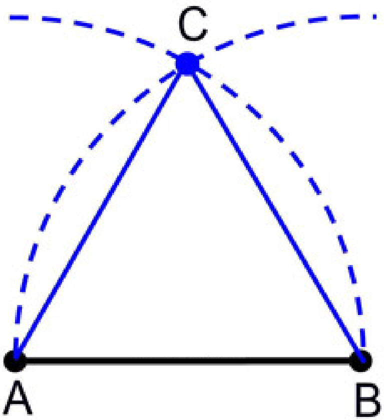
Or: Why 65537?

# The rules of the game



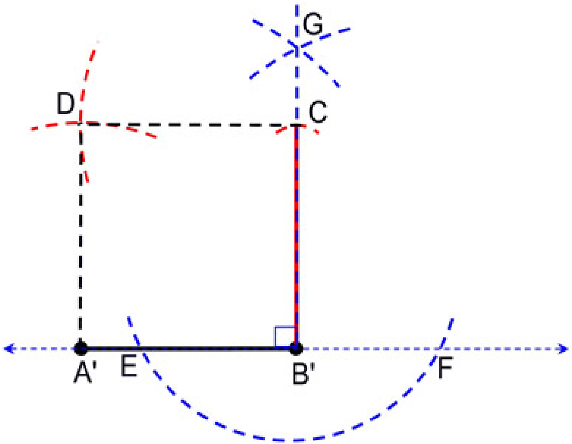
- Create lines using two existing points
- Create circles using two existing points
- Create points using intersections

# The triangle




- The construction of the triangle is easy and known for millennia
- Question What regular polygons can be constructed?

## The square



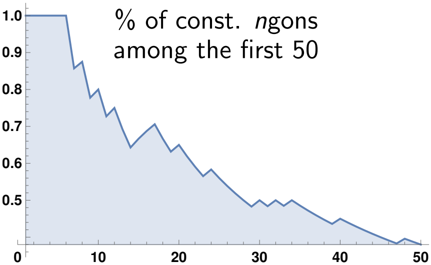

- The construction of the square is just a bit harder and known for millennia
- Ok, this problem is easy right?

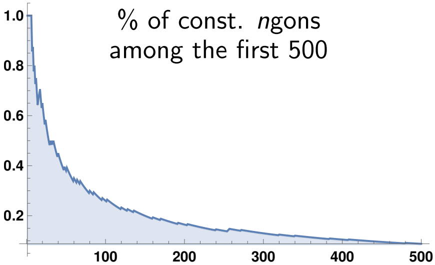
## Enter, the theorem

The regular *n*gon is constructible if and only if

$$n = 2^k p_1 \ldots p_t$$

- Here $p_i$ are distinct Fermat primes $p_i = 2^{2^j} + 1$
- The only known Fermat primes up to date are $3, 5, 17, 257, 65537$
- The 66537gon is possibly the biggest constructible elementary polygon
- So, almost no regular *n*gon is constructible



## An easy problem? Well...

# Ueber die Teilung des Kreises in 65537 gleiche Teile.

Von

**J. Hermes** in Lingen.

(Vorgelegt von F. Klein in der Sitzung am 5. Mai 1894.)

1) Bekanntlich erfordert die Gleichung:

$$r^p - 1 = 0 \text{ für } p = 2^{2^\mu} + 1$$

- The Gauss–Wantzel theorem is non-explicit
- The first explicit construction of the...
  - (a) ...regular 3 and 5gons are ancient
  - (b) ...regular 17gon is from 1796
  - (c) ...regular 257gon is from 1822
  - (d) ...regular 65537gon is from 1894 (after decades of work)

**Thank you for your attention!**

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I hope that was of some help.