What is...the small-world effect?
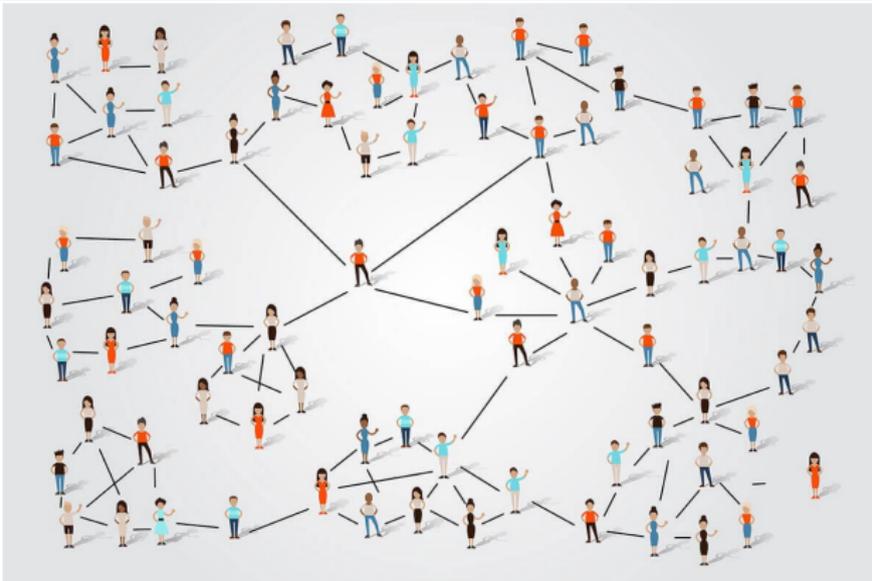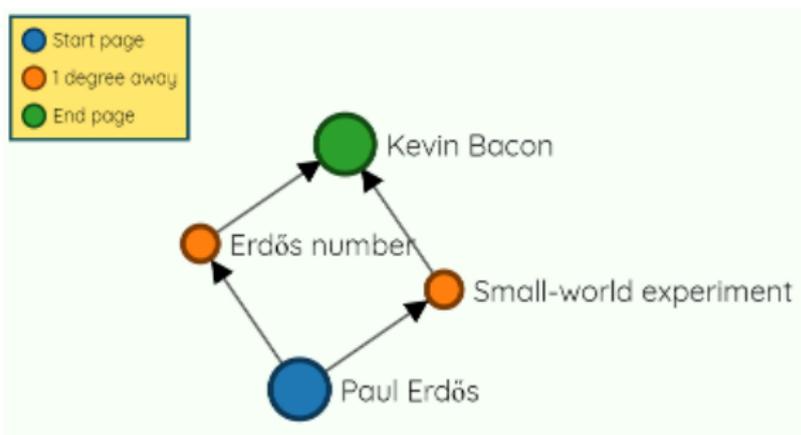
Or: Everyone knows everyone!?

Small-world experiments


- Original experiments (∼1967/69) "How many steps are humans separated?"

- Mails from Nebraska→Massachusetts ∼ 5.2 steps
  from Boston→Massachusetts < 5.2 steps

- The result suggested that human society is a small-world network

- How to address this mathematically ?

More small-world – the six degrees


▶ The small-world phenomenon has been rediscovered many times :

- Erdös number $+$ variations $=$ collaboration distance
- Wiki distance
- Various networks of brain neurons
- Many more

▶ What is special about small-world networks?
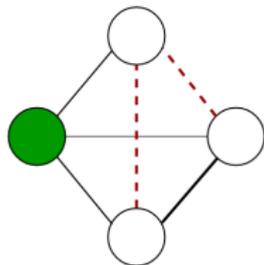
Small-world networks
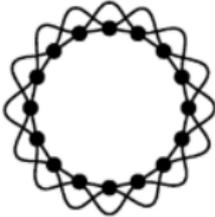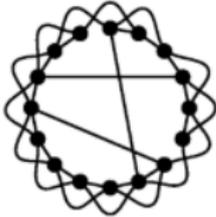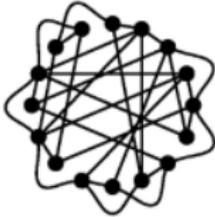
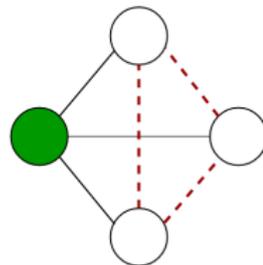|			
Network	Lattice, Ordered	Small World	Random, Disordered
Clustering Coefficient	High	High	Low
Mean Path Length	Long	Short	Short

► Small-world tend to have short distances

► Small-world tend to cluster My friends' friends tend to be my friends
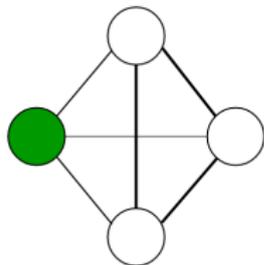
Enter, the definition (the definition is the theorem)

A small-world network is a graph G such that

Average path length $L(G) \approx L(\text{“random graph”})$

Local cluster coefficient $C(G) = \frac{2\times \text{adj. edges}}{(\text{number of neighbors } k)(k-1)} \gg C(\text{“random graph”})$
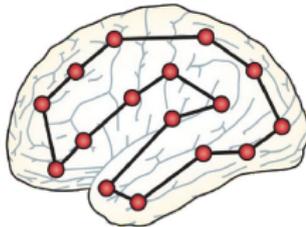
where “random graph” depends on the model used

Network	size	av. shortest path	Shortest path in fitted random graph	Clustering (averaged over vertices)	Clustering in random graph
Film actors	225,226	3.65	2.99	0.79	0.00027
MEDLINE co-authorship	1,520,251	4.6	4.91	0.56	1.8×10^{-4}
E.Coli substrate graph	282	2.9	3.04	0.32	0.026
C.Elegans	282	2.65	2.25	0.28	0.05

The brain, yet again
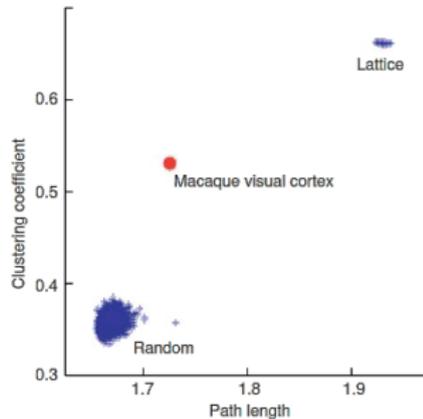
Regular network   Small-world network   Random network
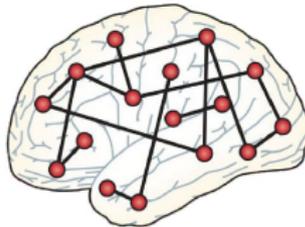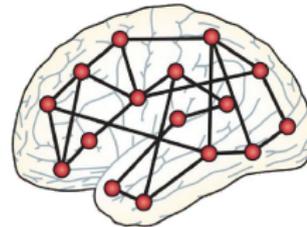
Local efficiency   **Optimal balance**   **Global efficiency**
Specific abilities   Broad abilities   General ability

A lot of brain networks are known to be small-world   Interpretation open

Thank you for your attention!

I hope that was of some help.