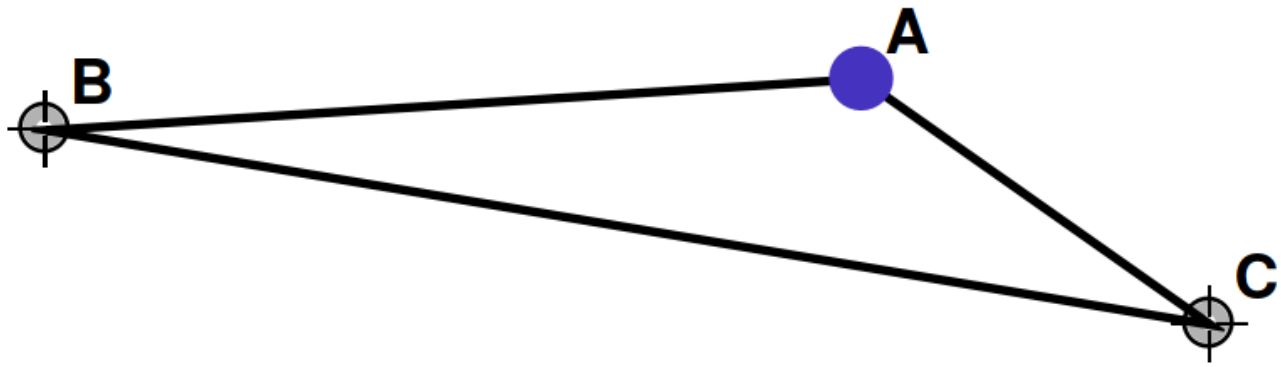
What are...the three geometries?

Or: 0, 1 and ∞

Angles in a Triangle

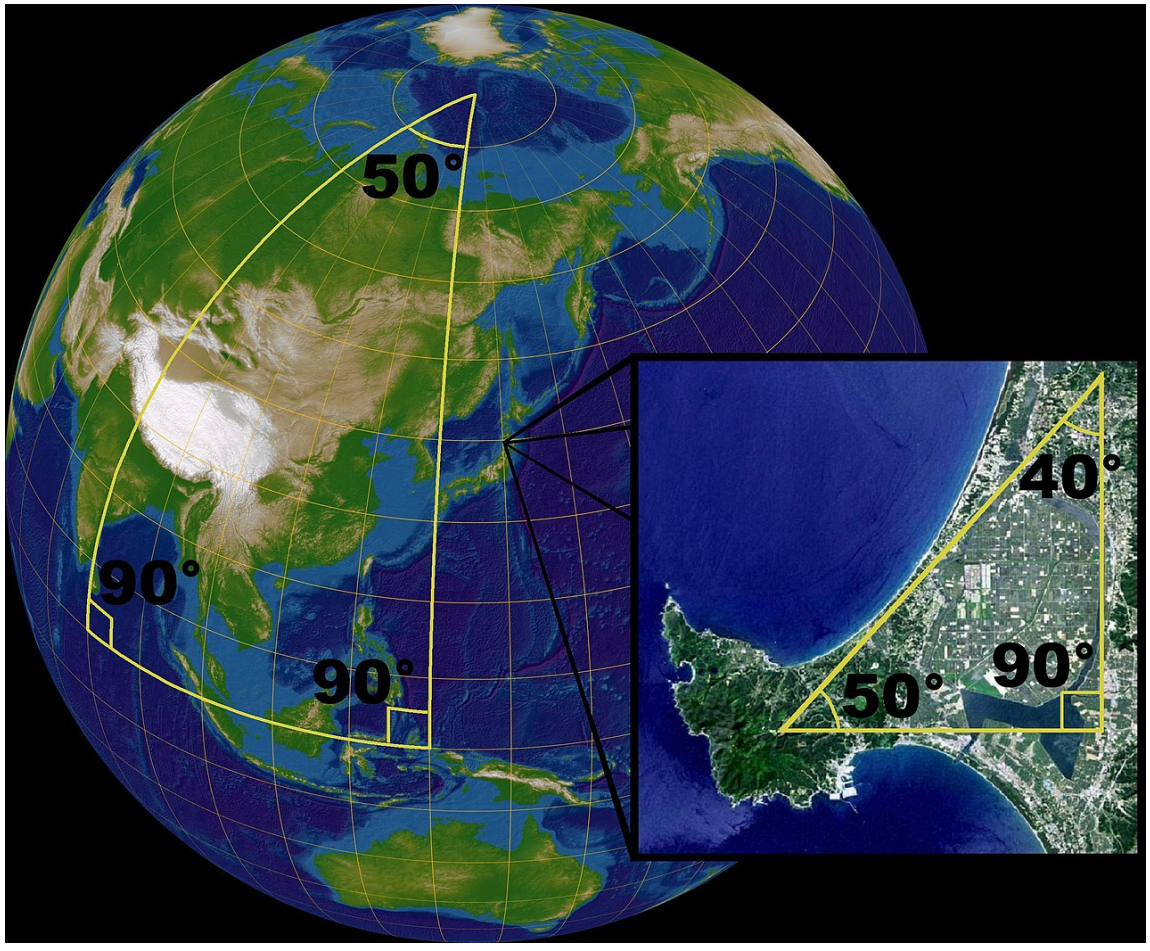
$m\angle CAB = 141.283^\circ$ | $m\angle ABC = 13.078^\circ$ | $m\angle BCA = 25.639^\circ$

$$m\angle CAB + m\angle ABC + m\angle BCA = 180.00^\circ$$



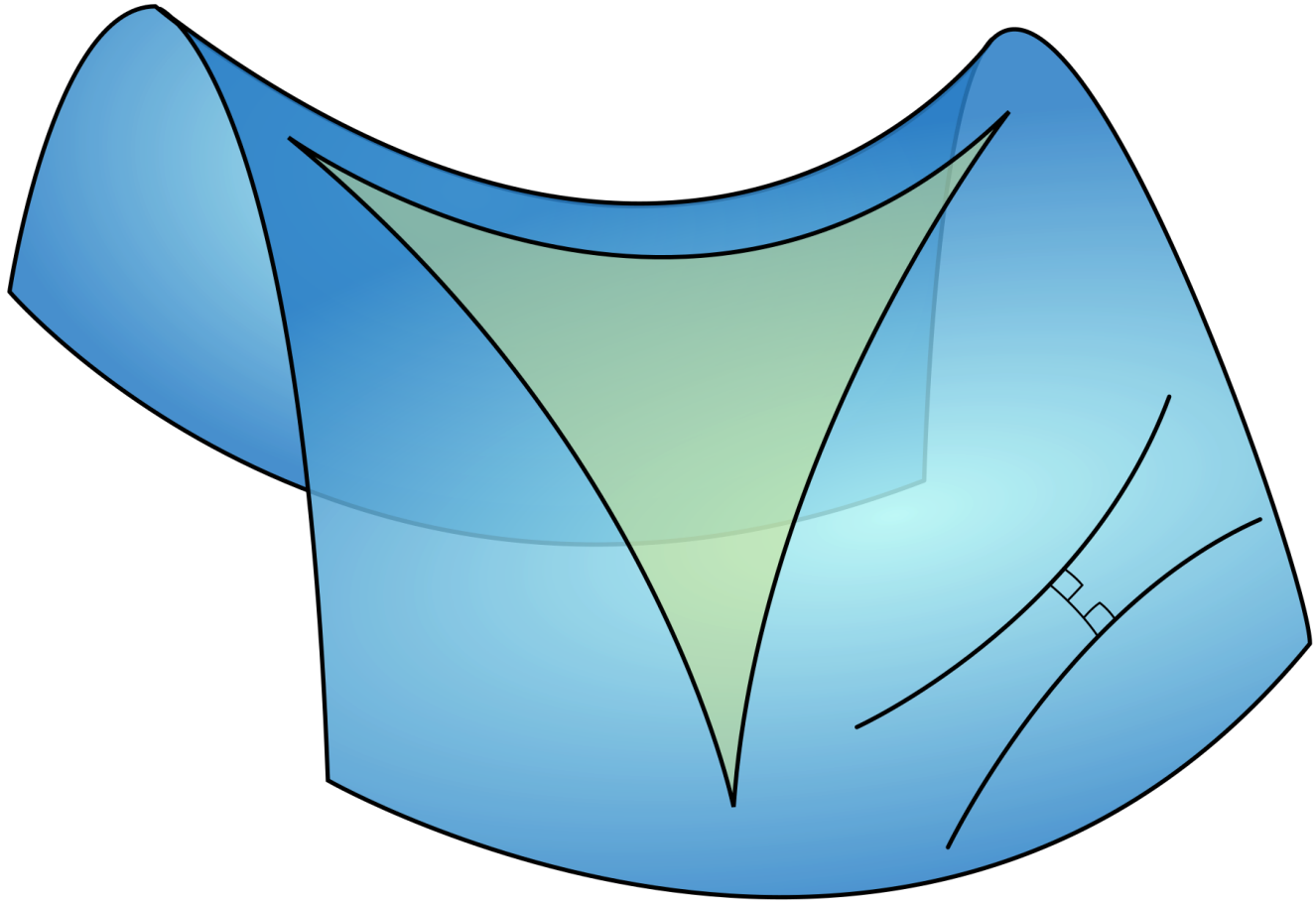
- EG is ancient More than 2000 years
- In EG all triangles have angles adding to 180° Normal triangles
- In EG there is exactly one parallel line

Spherical geometry (SG)



- SG is old, but not ancient 19th century
- In SG all triangles have angles adding to more than 180° Fat triangles
- In EG there are no parallel lines

Hyperbolic geometry (HG)

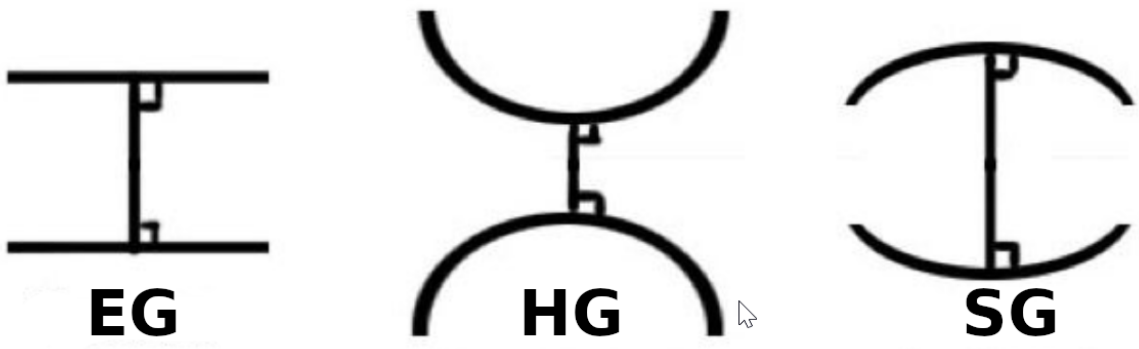


- HG is old, but not ancient 19th century
- In HG all triangles have angles adding to less than 180° Thin triangles
- In HG there are ∞ many parallel lines

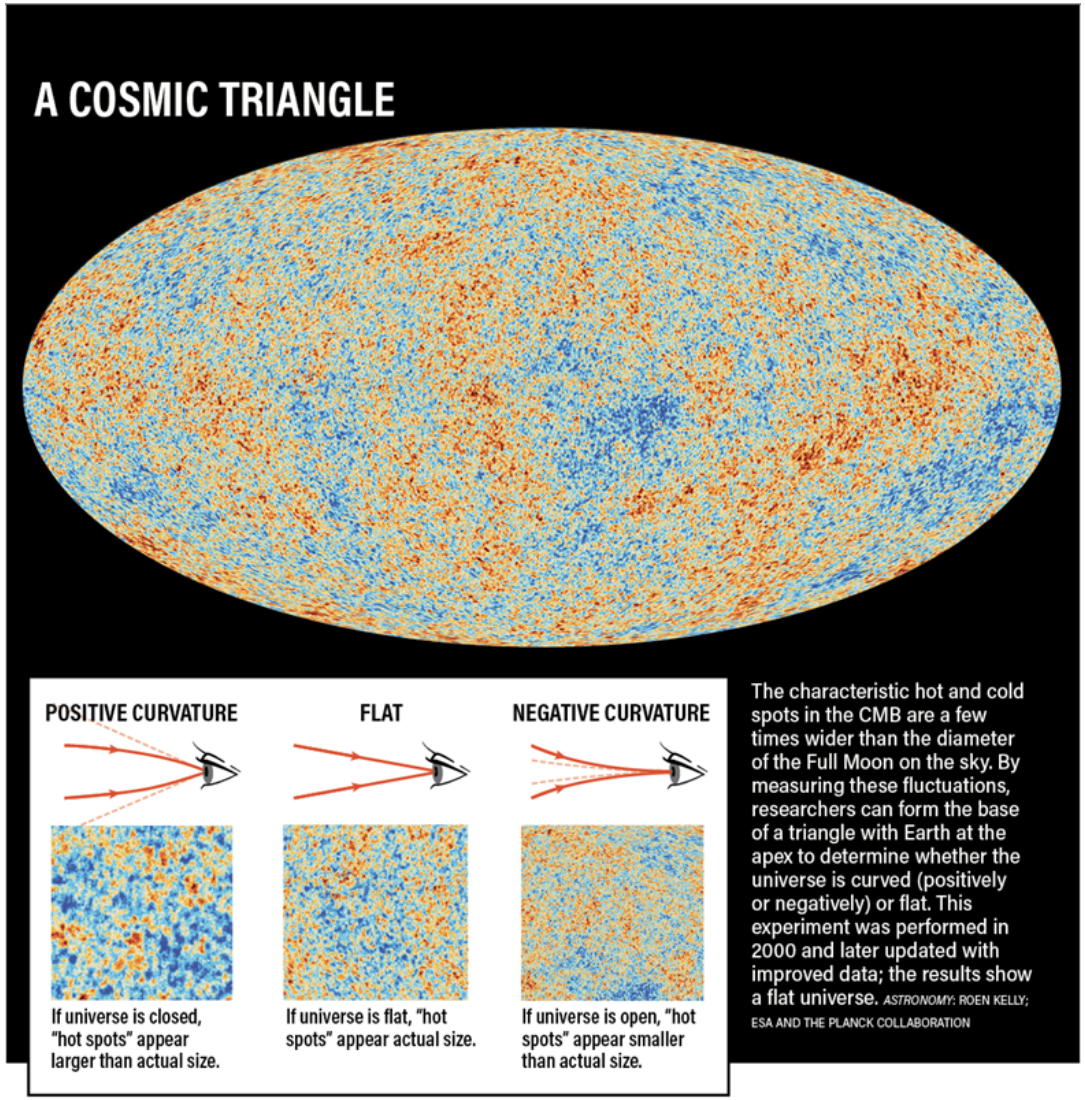
Enter, the theorem

There are only three geometries on surfaces, axiomatically given by:

- Line segments exists
- Infinite lines exist
- Circles exists
- All right angles are congruent
- A version of the parallel postulate
  - EG Through a point not on a given line L, there is one line not meeting L
  - SG Through a point not on a given line L, there is no line not meeting L
  - HG Through a point not on a given line L, there are ∞ many lines not meeting L



The shape of space



- It is not easy to determine the geometry we are living in
- The curvature of the universe is $\approx 1 \pm 0.1$ 1=EG, >1=SG, <1=HG

Thank you for your attention!

I hope that was of some help.