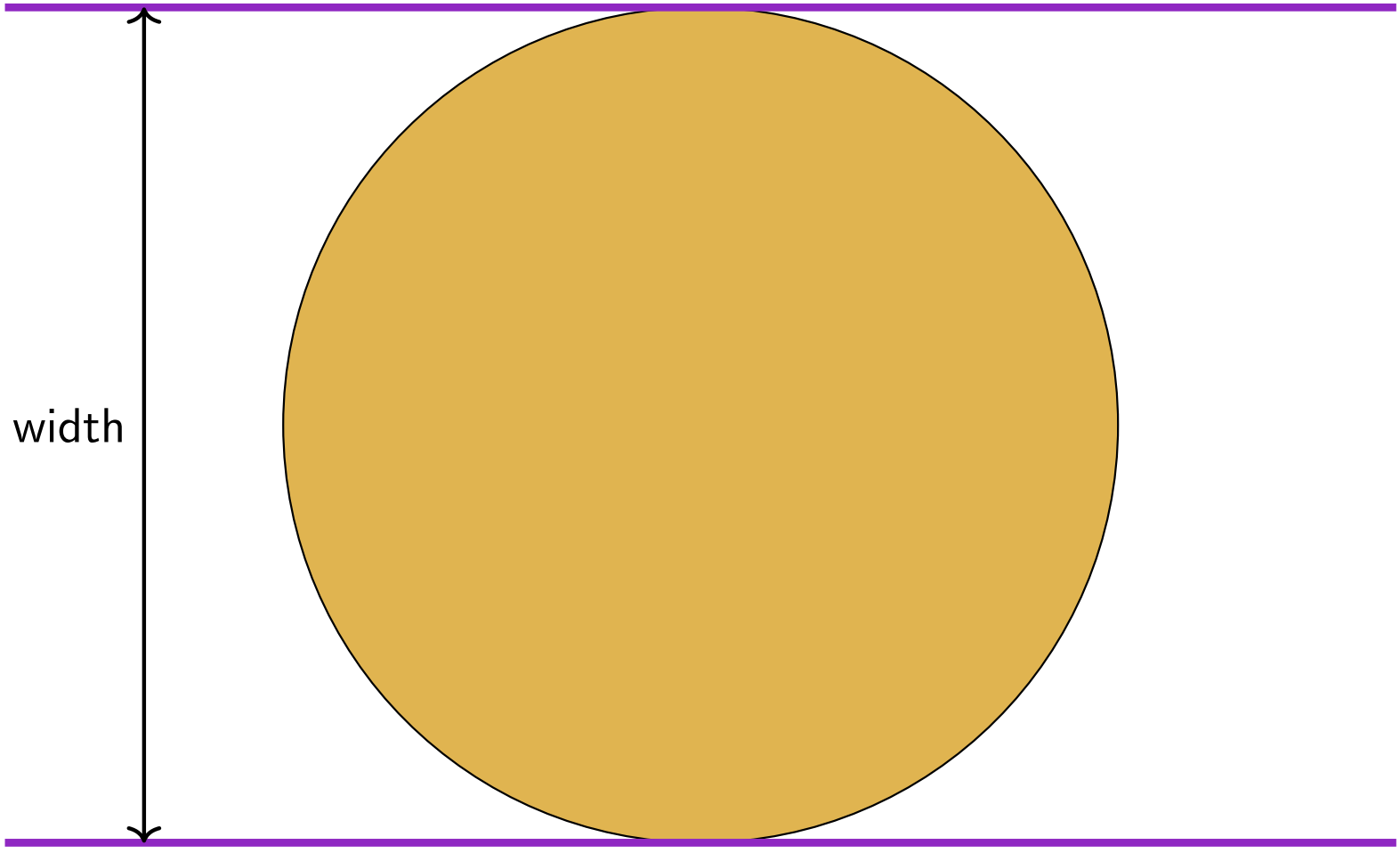
What is...roundness?

Or: My circle has corners!

A good definition?



- A circle is a 2d shape of constant width Sure!
- A two-dimensional shape of constant width is a circle Ah, sure?

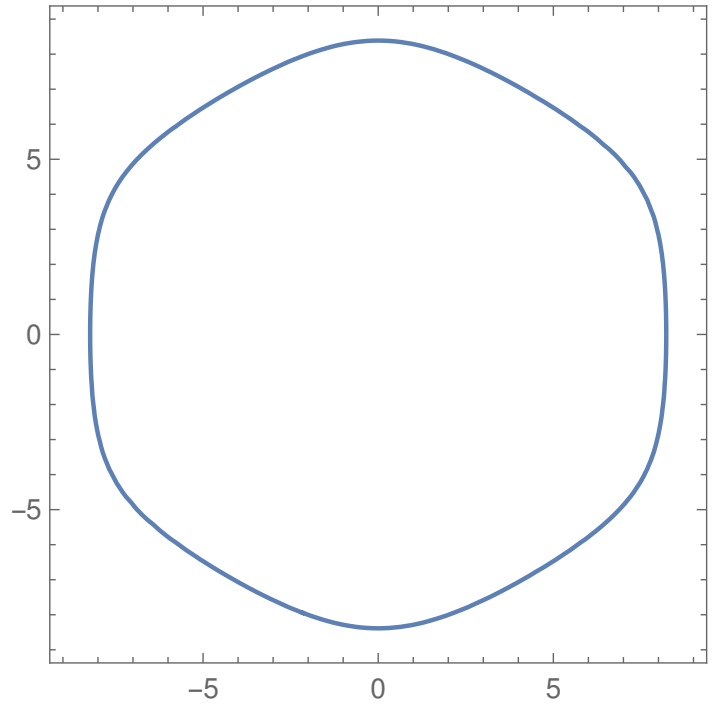
Beware Common engineering problems

- A 2d shape of constant width is rarely a circle
- The Australia 50cent coin is of constant width but definitely not a circle

How bad can it get? Pretty bad...

A Reuleaux triangle

A smooth example



- Even "A smooth 2d shape of constant width is a circle" is not true
- My favorite construction: Use polynomial equations to find these, *e.g.*

$$f(x,y)=0 \text{ with } f(x,y)=(x^2+y^2)^4-45(x^2+y^2)^3-41283(x^2+y^2)^2$$
$$+7950960(x^2+y^2)+16(x^2-3y^2)^3+48(x^2+y^2)(x^2-3y^2)^2$$
$$+x(x^2-3y^2)\left(16(x^2+y^2)^2-5544(x^2+y^2)+266382\right)-720^3$$

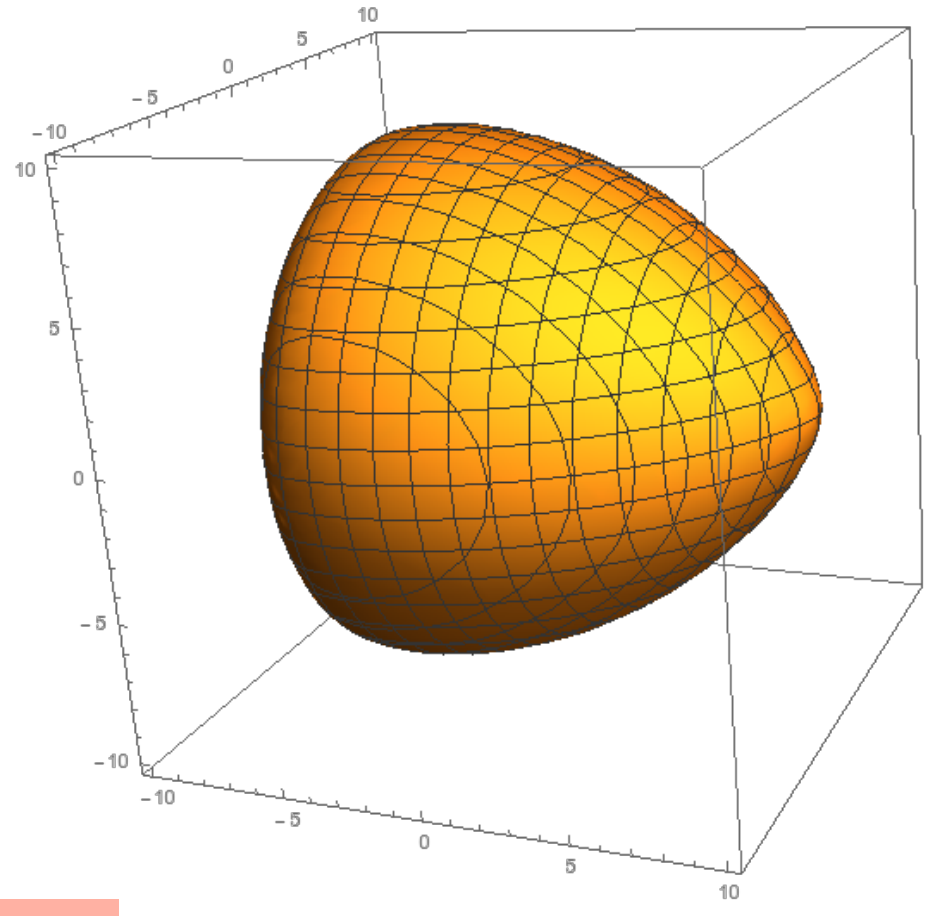
Enter, the theorem

Constant width can be defined *verbatim* in any dimension

- There exist objects of constant width in any dimension

```
In[2]:= ContourPlot3D[
  (x^2 + y^2 + z^2)^4 - 45 (x^2 + y^2 + z^2)^3 - 41283 (x^2 + y^2 + z^2)^2 +
    7950960 (x^2 + y^2 + z^2) + 16 (x^2 - 3 y^2 - 3 z^2)^3 +
    48 (x^2 + y^2 + z^2) (x^2 - 3 y^2 - 3 z^2)^2 +
    (x^2 - 3 y^2 - 3 z^2) x
     (16 (x^2 + y^2 + z^2)^2 - 5544 (x^2 + y^2 + z^2) + 266382) == 720^3,
  {x, -10, 10}, {y, -10, 10}, {z, -10, 10}]
```

Out[2]=

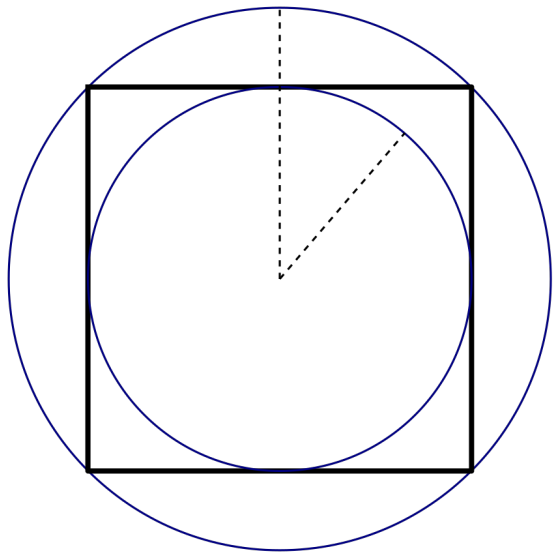


- Almost all (in a precise sense) are smooth
- Some are smooth and have trivial symmetry group

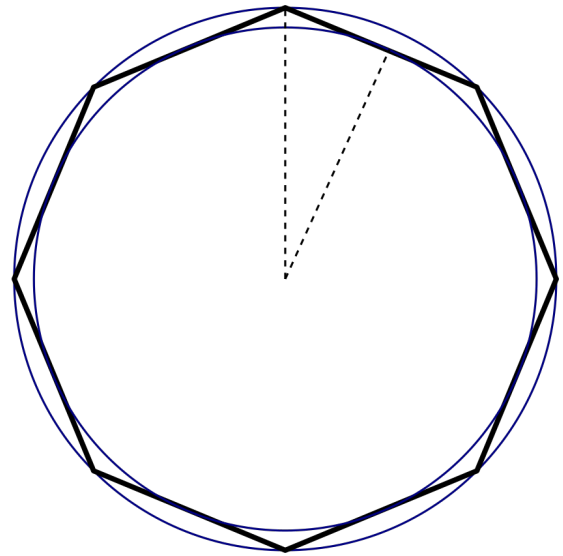
What is round?

The roundness $r(X)$ of a reasonable shape X (in 2d) is the ratio of radii:

$$r(X) = \frac{\text{inside fitting circle}}{\text{outside fitting circle}}$$



$r \approx 0.7$



$r \approx 0.9$

- $(r(X) = 1) \Rightarrow$ (Constant width), but $\not\Leftarrow$
- Determining the roundness in practice is an important problem in engineering
- There is a similar definition in any dimension

Thank you for your attention!

I hope that was of some help.