What is...the Kuratowski–Wagner theorem?
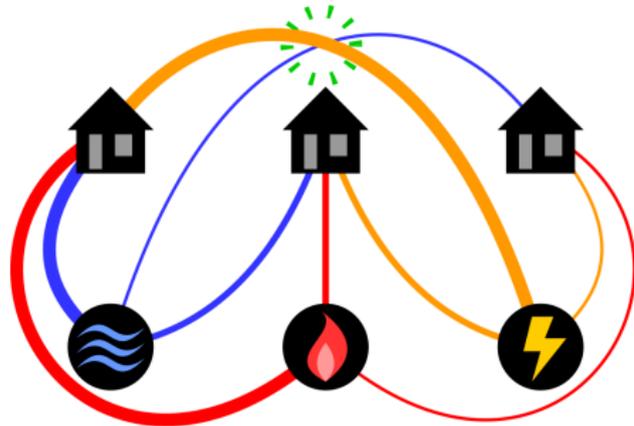
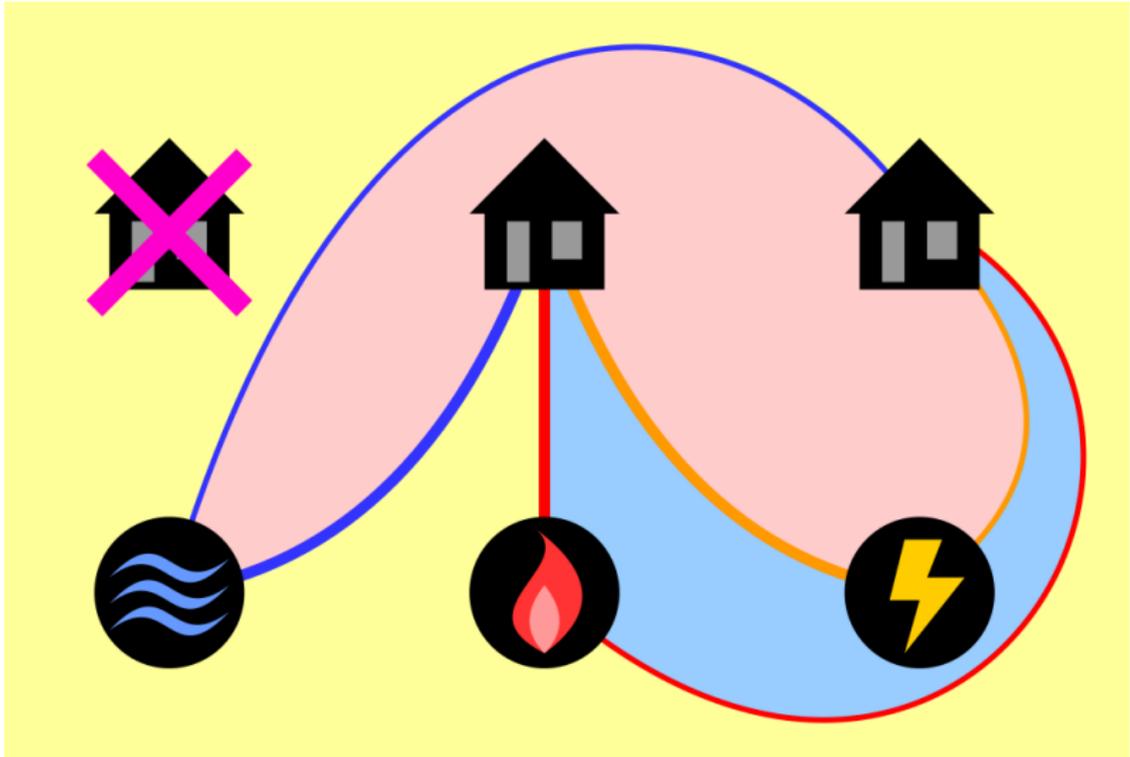Or: Forbidden pieces

Water, gas and electricity

The setup

non-solution

Question Is there a way to connect all three houses to water, gas and electricity without the connections crossing each other?
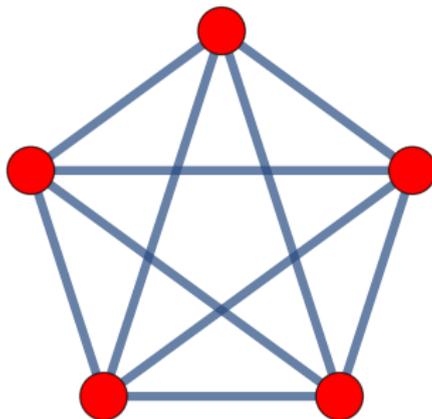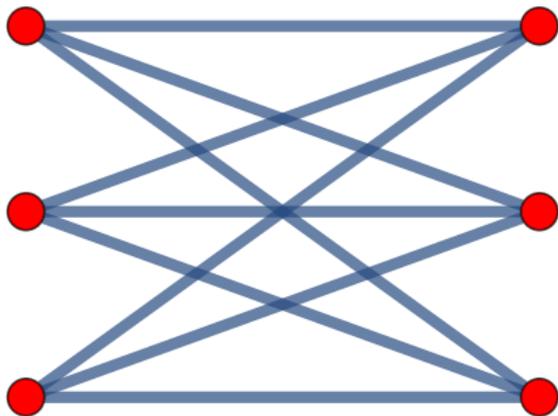
Proof without word (The electricity is stuck!)

Planar graphs

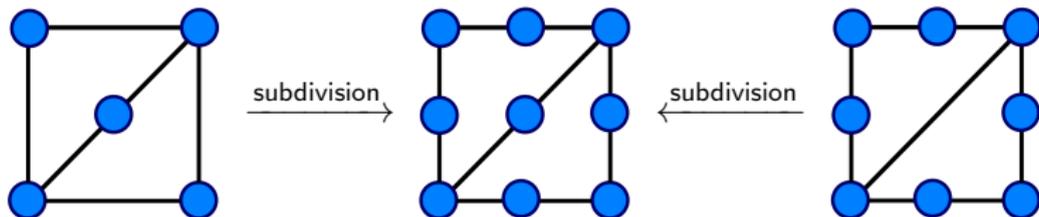Complete bipartite graph on 6 vertices $K_{3,3}$ Complete graph on 5 vertices K_5

▶ A graph is an abstract object living nowhere
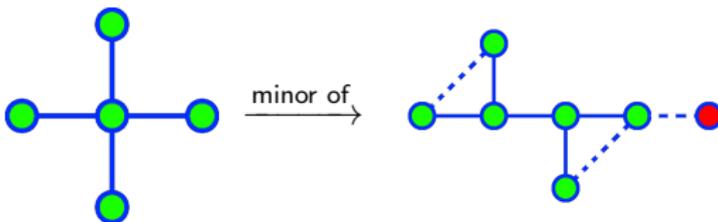
▶ A graph is planar if it can be embedded in the plane

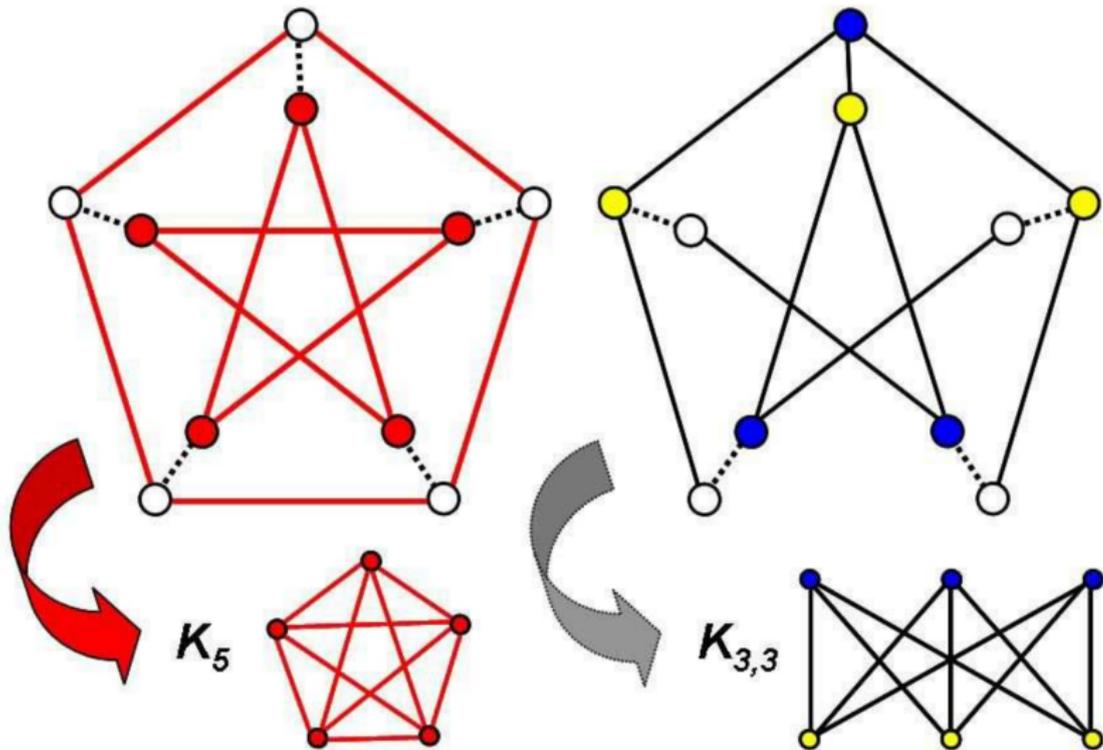Problem Given an abstract graph, how can you decide whether it is planar?

Enter, the theorem

A graph is planar if and only if it does not contain a subgraph which is a subdivision of $K_{3,3}$ or K_5

A graph is planar if and only if it does not contain $K_{3,3}$ or K_5 as a minor

The Petersen graph is not planar

K_5

$K_{3,3}$

▶ The Petersen graph contains a subdivision-subgraph of $K_{3,3}$ — Not planar

▶ The Petersen graph contains a minor K_5 — Not planar

Thank you for your attention!

I hope that was of some help.