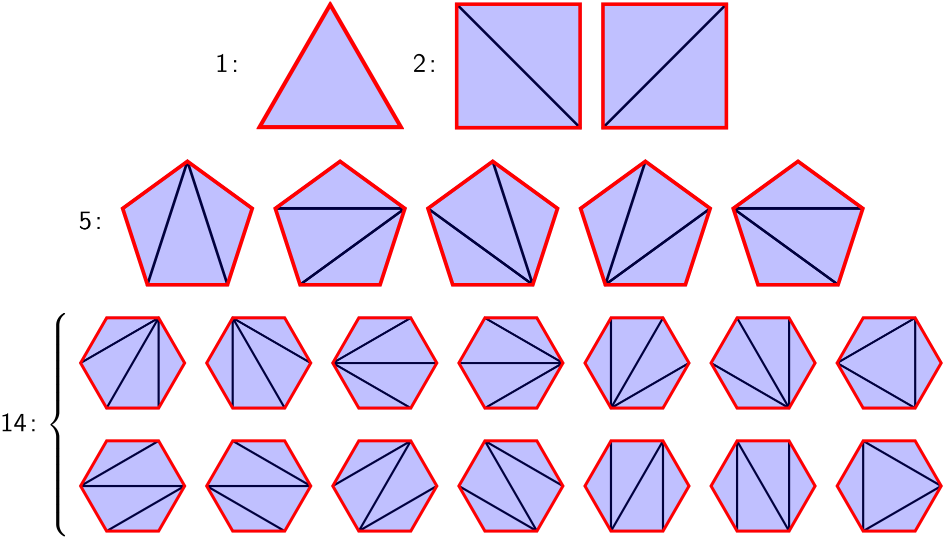
What are...Catalan numbers?

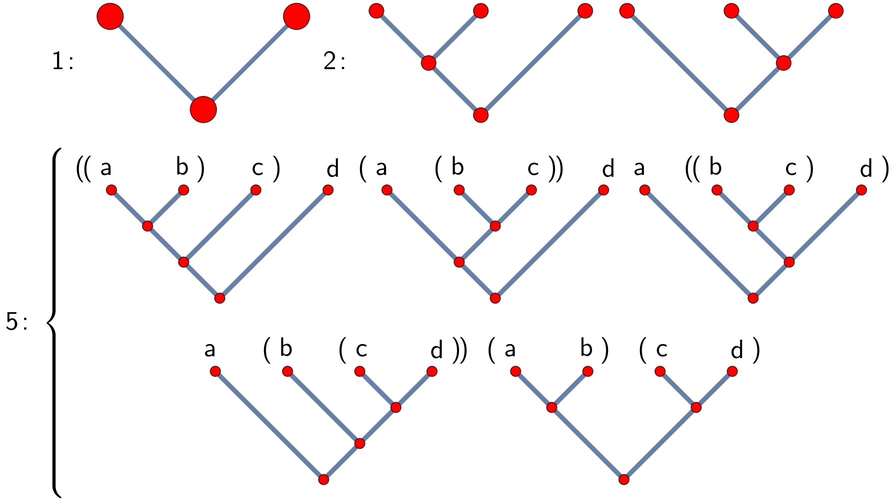
Or: In praise of counting

How many ways are there to cut a regular *n*-gon into triangles?



Answer There are $C_n = \frac{1}{n+1}\binom{2n}{n}$ many: $1, 2, 5, 14, \ldots$

How many ways are there to bracket $n-1$ symbols?



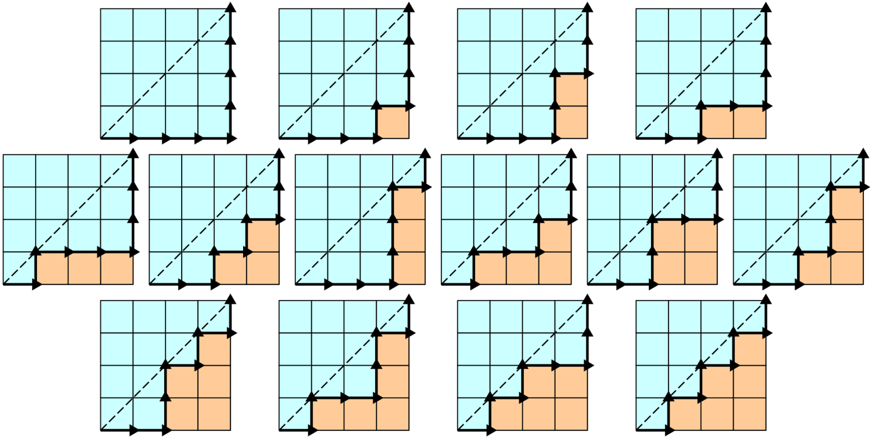
14: homework ;-)

Answer There are $C_n = \frac{1}{n+1}\binom{2n}{n}$ many: $1, 2, 5, 14, \ldots$

How many ways are there to not cross the diagonal in a $n \times n$ grid?

$1, 2, 5$: homework ;-)

14:



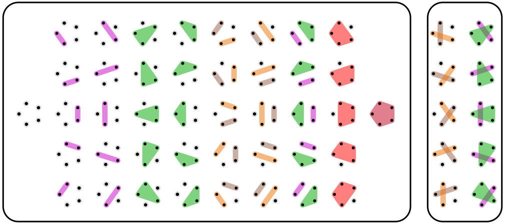
Answer There are $C_n = \frac{1}{n+1}\binom{2n}{n}$ many: $1, 2, 5, 14, \ldots$

Enter, the theorem

The Catalan numbers $C_n = \frac{1}{n+1}\binom{2n}{n}$ count, among other things, the following:

- What we have seen
- C_n = number of noncrossing partitions of $\{1, ..., n\}$

42:



- C_n = number of shuffles of $\{1, ..., n\}$ with no 3-term increasing subsequence
- C_n = number of Dyck words of length $2n$
- Much more, see *e.g.* OEIS

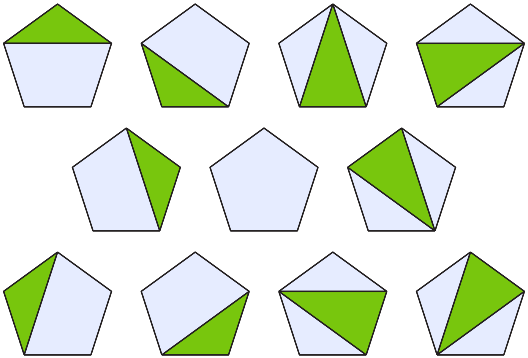
This is probably the longest entry in the OEIS, and rightly so

Quote from the OEIS entry A000108 (Catalan numbers, of course)

Schröder–Hipparchus a.k.a. super Catalan number

1, 1, 3, 11, 45, 197, 903, 4279, 20793, 103049, 518859, 2646723, 13648869, 71039373, 372693519, 1968801519, 10463578353, 55909013009, 300159426963, 1618362158587, 8759309660445, 47574827600981, 259215937709463, 1416461675464871

These numbers count again many things, for example subdivisions:



Thank you for your attention!

I hope that was of some help.