## What is...the art gallery theorem?

Or: How many cameras suffice?

How many cameras are needed to guard a museum?


- Museum $=$ closed polygon $P$
- Cameras $c$ are fixed points in $P$ and can see in any direction
- We want the whole of $P$ to be covered

- In this case $n=3 m$
- $n / 3$ cameras are needed
- $\lfloor n / 3\rfloor$ is definitely a lower bound

You might be able to do better - but that is not the question


- In this case $n=35$
- 4 cameras suffice
- 4 is of course much smaller than $\lfloor n / 3\rfloor=11$ but we do not case


## Enter, the theorem

$\lfloor n / 3\rfloor$ cameras always suffice but you can do better:

$$
\begin{gathered}
n=11 \\
\lfloor n / 3\rfloor=3
\end{gathered}
$$



- The theorem is due to Chvátal $\sim 1975$
- The proof by Fisk ~1978 uses graph theory Let us sketch how that works!


## A problem in graph theory



- Every museum admits a triangulation Easy but not obvious
- Every triangulation can be tricolored Easy but not obvious
- At least one of the color classes contains at most $\lfloor n / 3\rfloor$ vertices Harvest



## Thank you for your attention!

I hope that was of some help.

