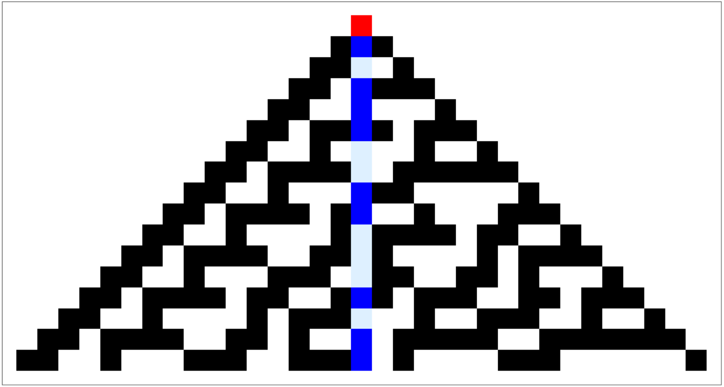
What are...random numbers?

Or: Compressible?

Algorithms generate randomness

rule 30:



random number generated:

$$1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-5} + 0 \times 2^{-6} + 0 \times 2^{-7} + 1 \times 2^{-8} + 1 \times 2^{-9} + \ldots =$$

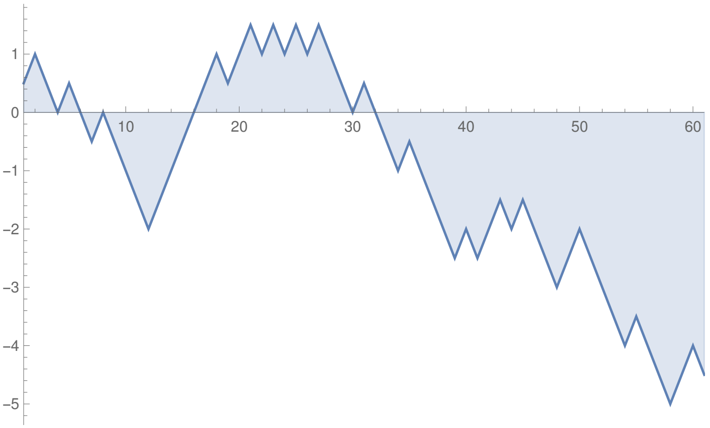
0.724777221679688

Mathematica generated once random numbers using rule 30

But no computer can generated "true randomness"

π generates randomness

$\pi \approx 11.001001000011111101101010100010001000010110100011_2$



The digits of π have the behavior of a random walk

But no computable real number can generated “true randomness”

Non-random numbers

The number

$$\underbrace{1...1}_{10000 \text{ times}}$$

is not random because it can be compressed into

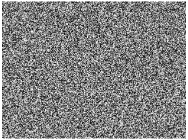
```
for i=1 to 10000 do
  print i
```

compressible : *abababababababababababababababab*	string length 32
⇝ "write ab 16 times"	string length 17
not compressible : *4c1j5b2p0cv4w1x8rx2y39umgw5q85s7*	string length 32
⇝ "write 4c1j5b2p0cv4w1x8rx2y39umgw5q85s7"	string length 38

Enter, the theorem/philosophy!

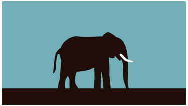
Kolmogorov randomness. A string is random if any computer program that can produce that string is at least as long as the string itself

- This is not the formal definition (see links in the description)
- All reasonable choices of programming language work the same way
- This applies to real numbers $w = a_1 a_2 a_3 ...$
- Random real numbers form a measure 1 subset of reals, non-random numbers a measure 0 subset Almost all real numbers are random
- These ideas are used in data compression:



Start: 43'571 bytes

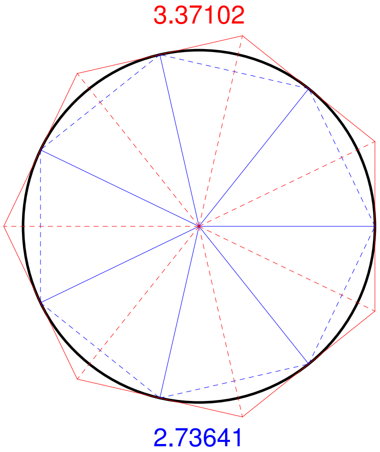
Compressed: 43'794 bytes



Start: 8'853 bytes

Compressed: 7'539 bytes

π is not random



Archimedes' approximation of π is slow. Better: use something like

$$\frac{1}{\pi} = 12 \sum_{k=0}^{\infty} \frac{(-1)^k (6k)!(13591409 + 545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}}$$

to shows that π is not random

Thank you for your attention!

I hope that was of some help.