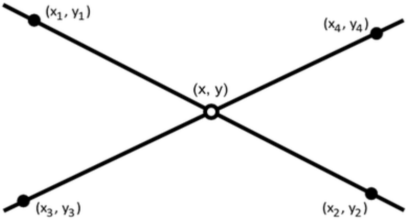
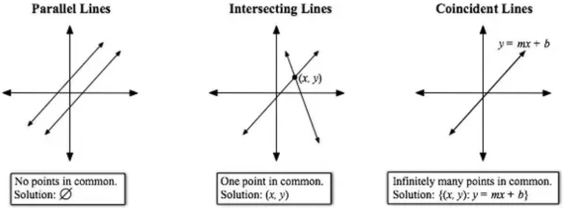
What is...the Cayley–Bacharach theorem?

Or: Nine and nine!

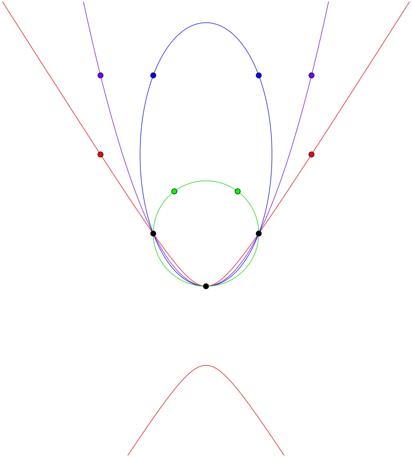
Two and one

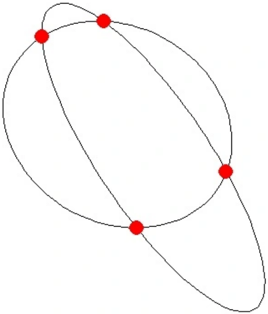




- 2 points determine a line
- Two unequal lines intersect in 1 point
- Let us do everything in projective geometry to avoid special cases

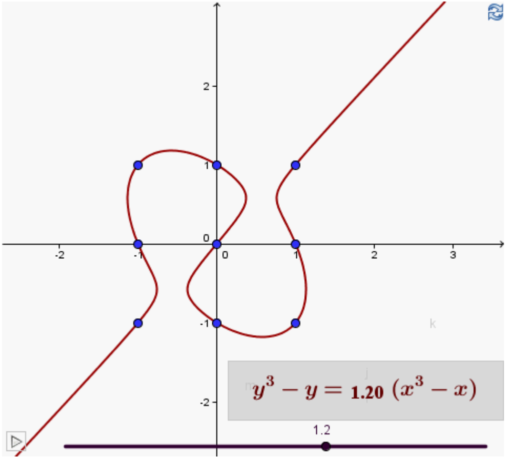
Five and four

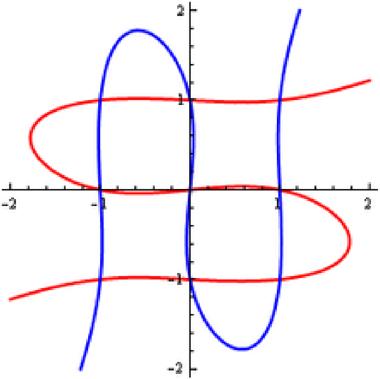




- 5 points determine a conic (e.g. circle, ellipse, parabola, hyperbola)
- Two unequal conics intersect in 4 point
- Conic = two variables X, Y + order 2 equation e.g. $X^2 + Y^2 = 1$, $X = Y^2$

Nine and nine

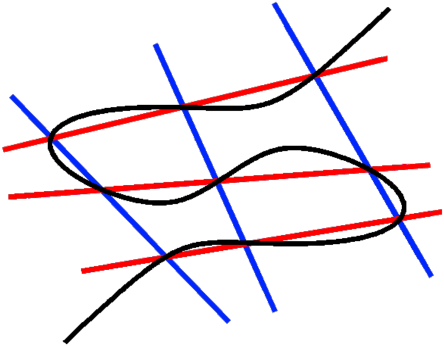


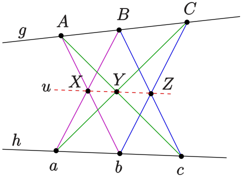


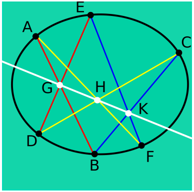
- 9 points determine a cubic (e.g. an elliptic curve)
- Two unequal cubics intersect in 9 point
- Cubic = two variables X, Y + order 3 equation e.g. $X^3 + Y^3 = 1$, $X^3 - X = Y^2$

Enter, the theorem

The intersection of two cubics determines another cubic



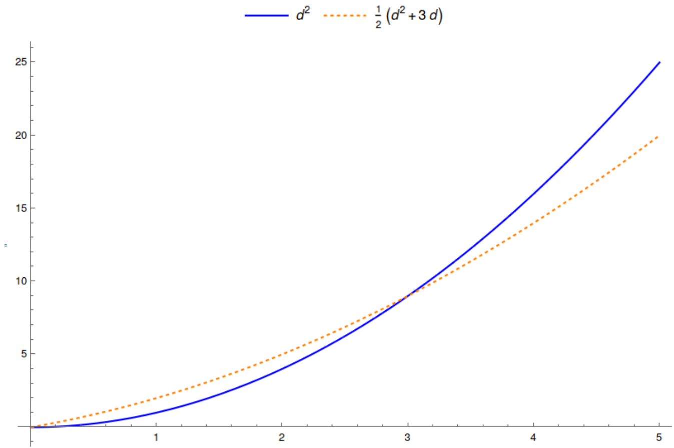




To be formally correct add "generic", "projective" and "algebraically closed"

- Above two of the cubics are unions of lines e.g. $(x-y-1)(x-y-2)(x+y-3)$
- Special cases are Pappus' theorem and Pascal's theorem (above right)

On growth rates



- For degree d curves, the number of intersection points is d^2
- For degree d curves, the number of points determining it is $1/2(d^2+3d)$
- So the theorem is essentially saying $d^2 = 1/2(d^2+3d) \Leftrightarrow d = 0, 3$

Thank you for your attention!

I hope that was of some help.