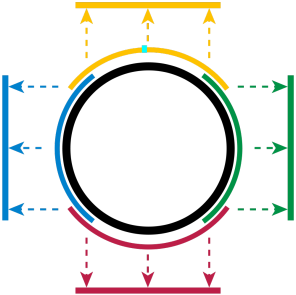
What is...the 3d Poincaré conjecture?

Or: Its a sphere? Its a sphere!

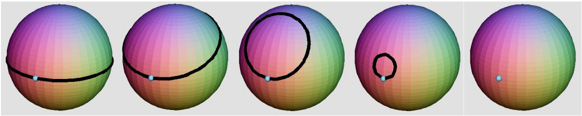
Locally discs

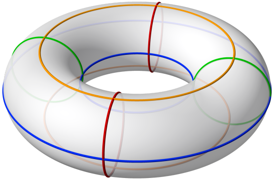


discs in 3d and 2d:

- n dim manifold (without boundary) = every point has an n dim disc D^n neighborhood + technical condition
- Closed = it fits into some big enough sack/ball
- Example The only closed 1 dim manifold is the circle

The 2 dimension case





- 2 dim manifolds = surfaces
- Classifying surfaces (up to homeomorphism) has a nice answer
- Observation The only closed sc surface is a sphere = soccer ball
- Simply-connected (sc) = every curve can be shrunk to a point

Poincaré and dimension three

CINQUIÈME COMPLÉMENT À L'ANALYSIS SITUS.

Par M. H. Poincaré, à Paris.

Adunanza del 22 novembre 1903.

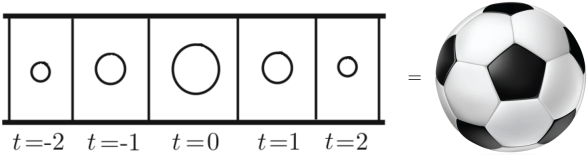
Il resterait une question à traiter :

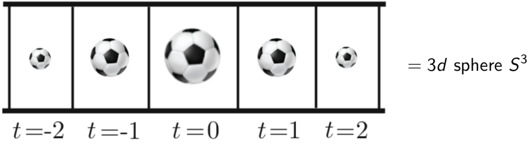
Est-il possible que le groupe fondamental de V se réduise à la substitution identique, et que pourtant V ne soit pas simplement connexe ?

- Closed 3 dim manifolds need four-space to be realized, so are hard to imagine
- Poincaré ∼1904 : classification in 3d is difficult, but maybe:
- Question The only closed sc 3 dim manifold is a sphere?

Enter, the theorem

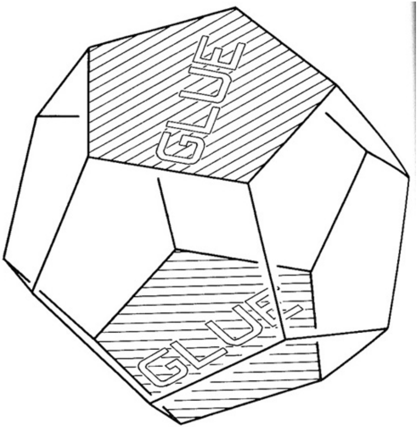
The answer to Poincaré's question is Yes!

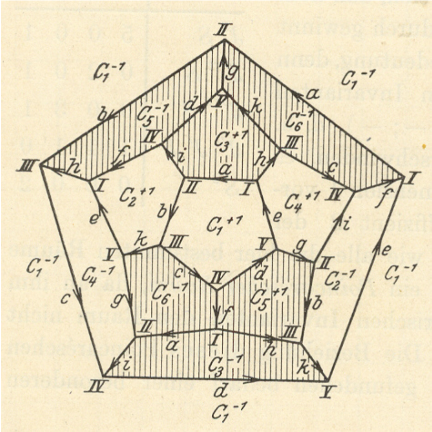




- The above is a theorem of many people, finalized by Perelman ∼2002
- The >3 dim analog was known for some time due to many people, e.g. Smale ∼1961 for >4 and Freedman ∼1982 for $=4$
- The smooth 4d version is "the last person standing in geometric topology"

Poincaré revised their question





- The original "Poincaré conjecture" was homology detects the 3-sphere
- Poincaré found a counterexample ∼1904 (later reformulated as "gluing opposite sides of a dodecahedron") and then changed the "conjecture"
- Maybe this is why it was carefully called a question and not a conjecture

Thank you for your attention!

I hope that was of some help.