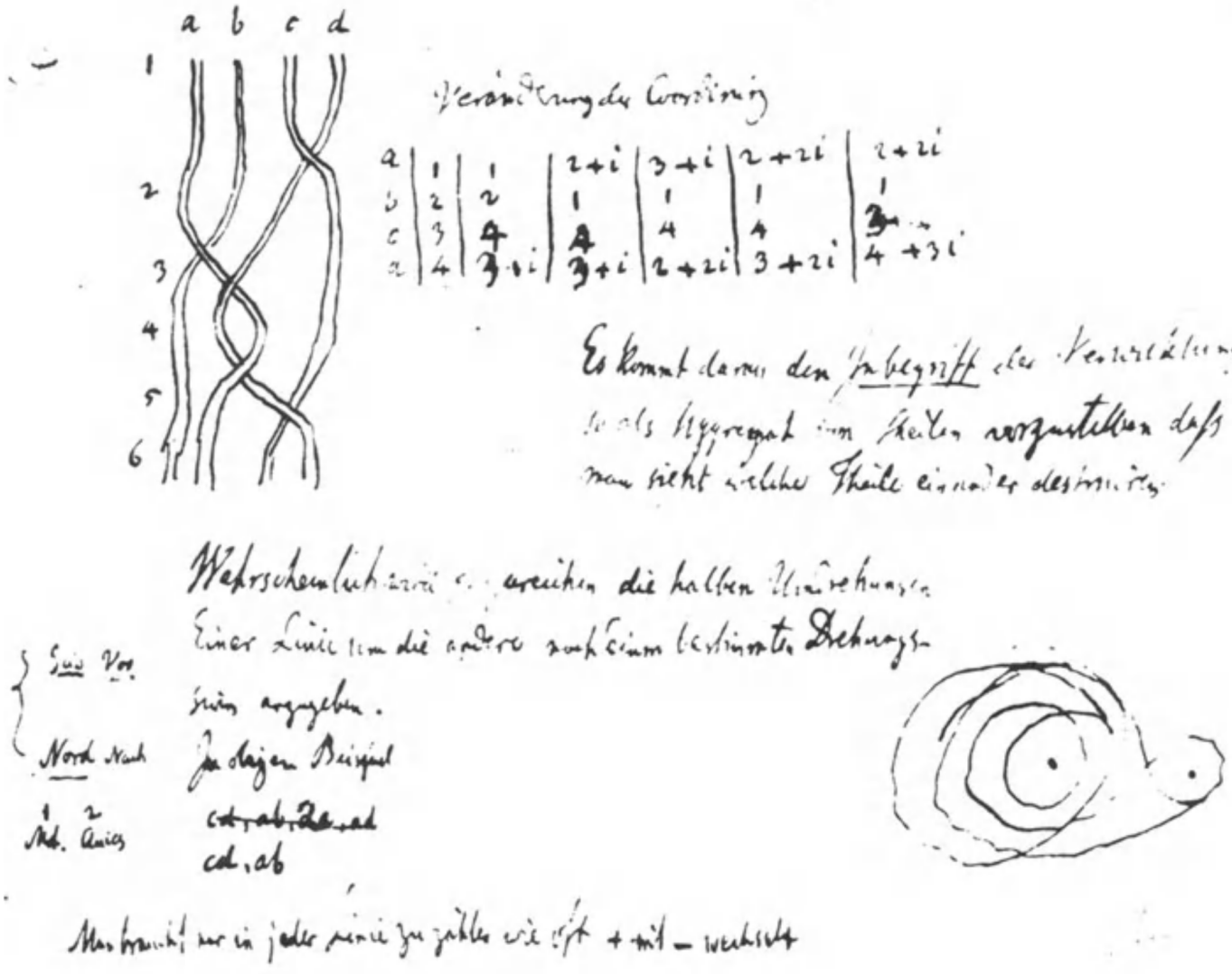
What are...Artin braid groups?

Or: Topology *via* algebra

Braids in mathematics?

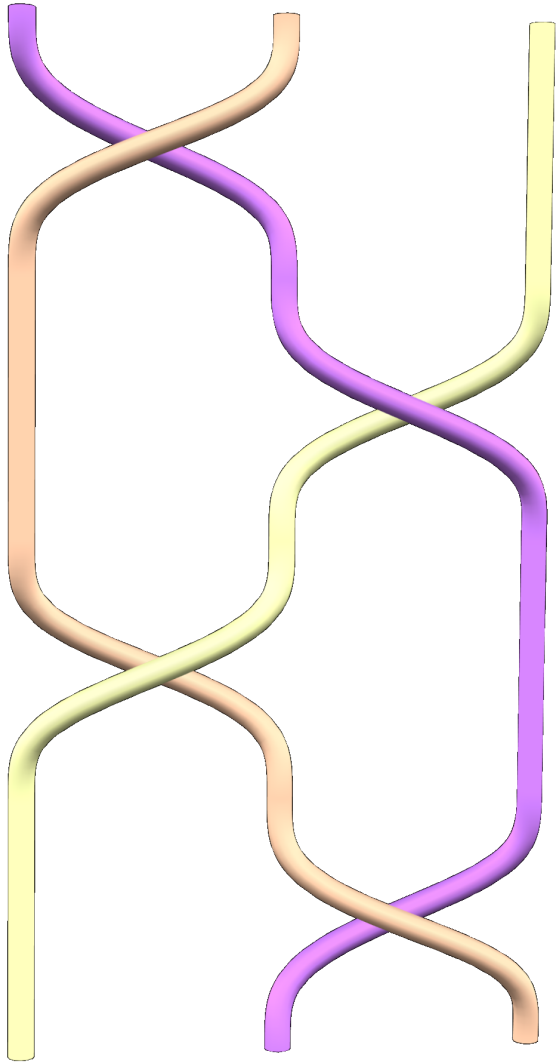
Braids are around for millennia, but how to study them mathematically?



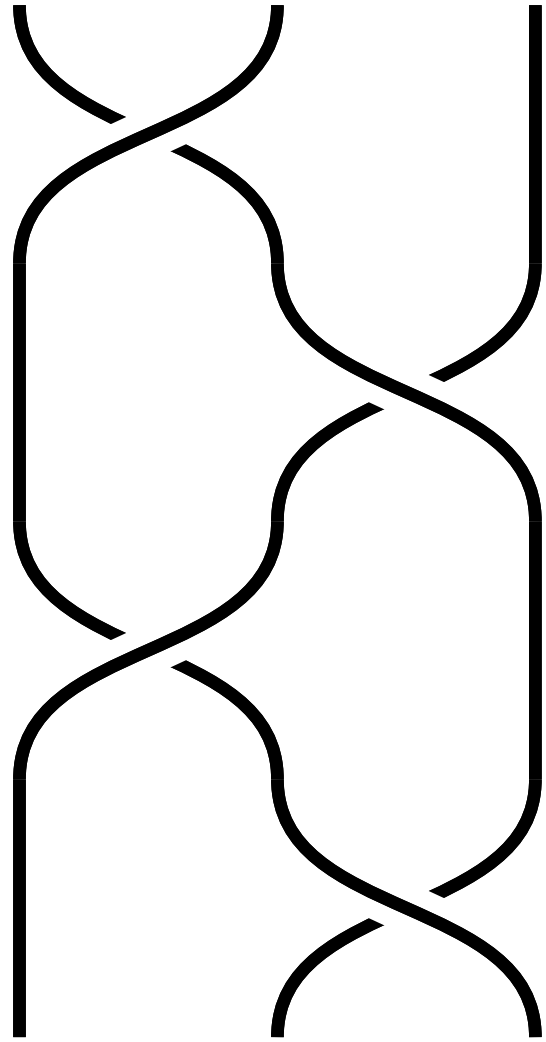
Gauss' handwritten notes ∼1820: first appearance of braids in mathematics?

Braids in 2d

$3d$:



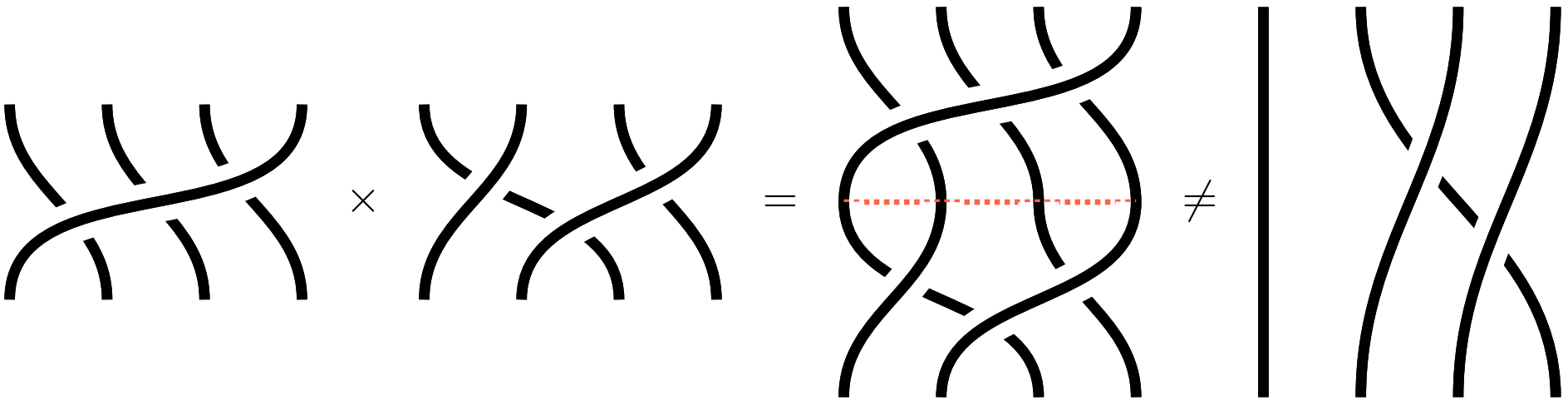
$2d$:



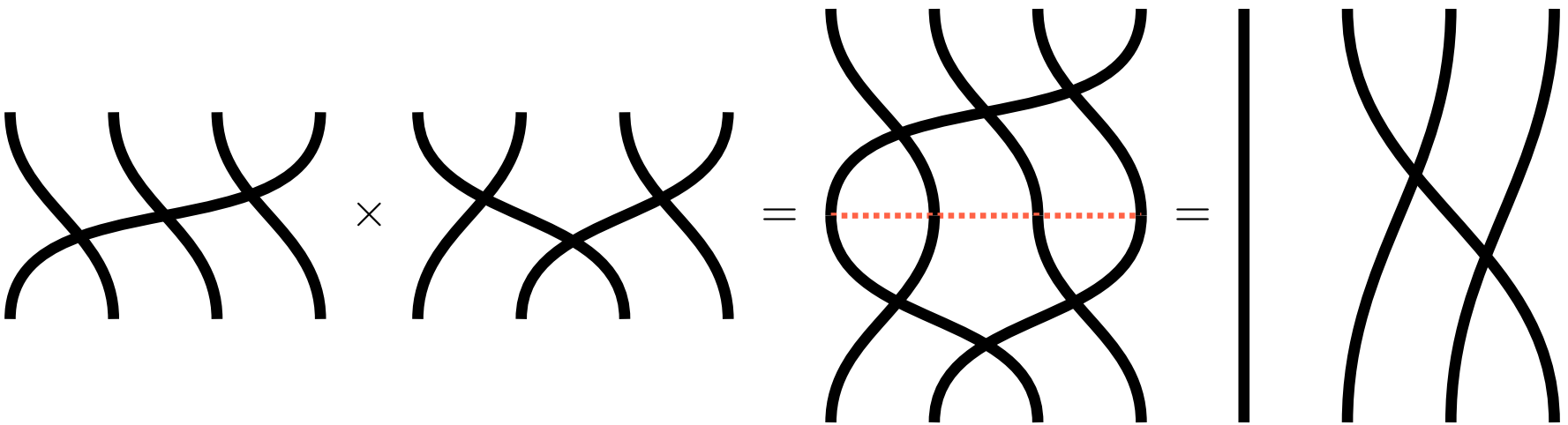
Question. Can one describe the information loss from 3d to 2d?

Its a group! gh **is "stack** g **on top of** h**"**

Braid group (infinite unless you just have one strand):



Symmetric group (always finite):

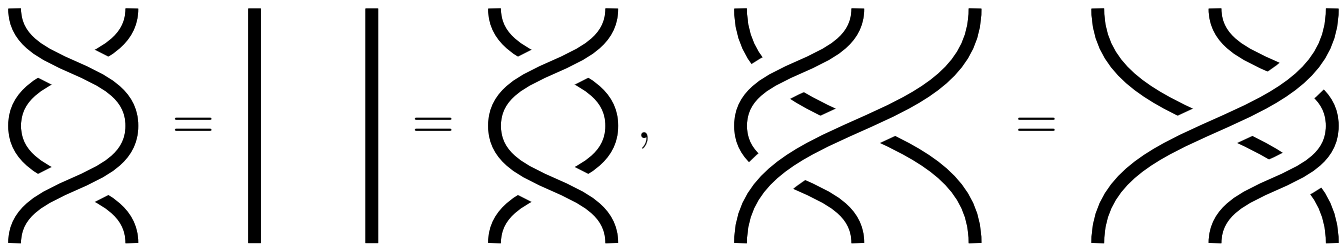


Enter, the theorem!

(a) Braids (topology) on n strands form a group Br_n (algebra)

(b) The group Br_n is generated by

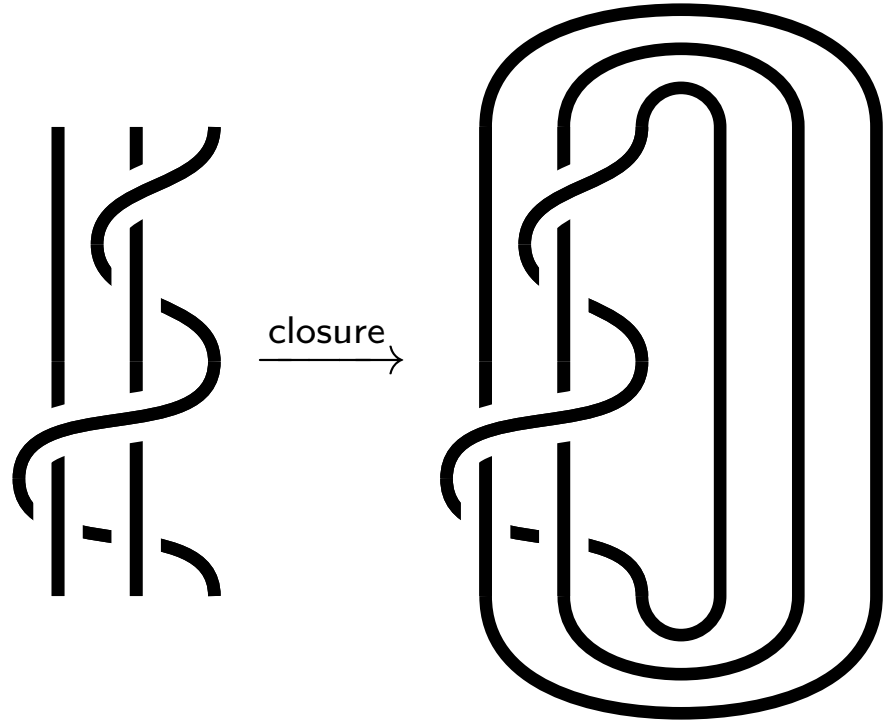
(c) Two elements in Br_n represent the same braid if and only if they are related by height moves or



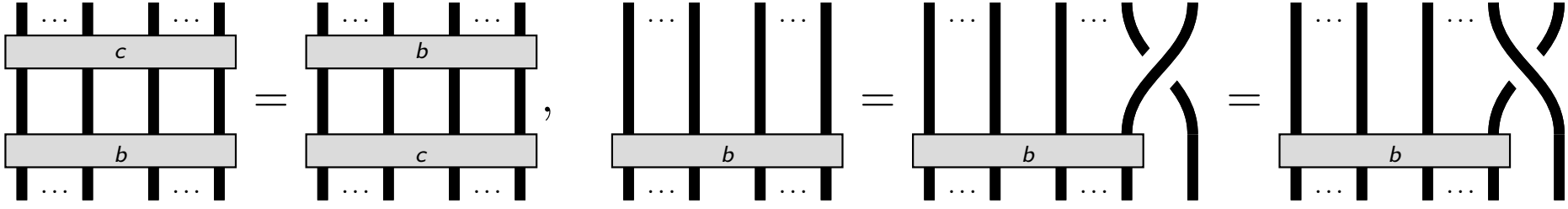
Consequences.

- One gets a purely algebraic way to study braids
- The symmetric group is a quotient, so one also gets a presentation for it

A purely algebraic way to study knots/links



- Alexander theorem. Every knot/link arises in this way
- Markov theorem. Two closures represent the same knot/link if and only if they are related by braid operations or



Thank you for your attention!

I hope that was of some help.