# What are...alpha shapes and complexes?

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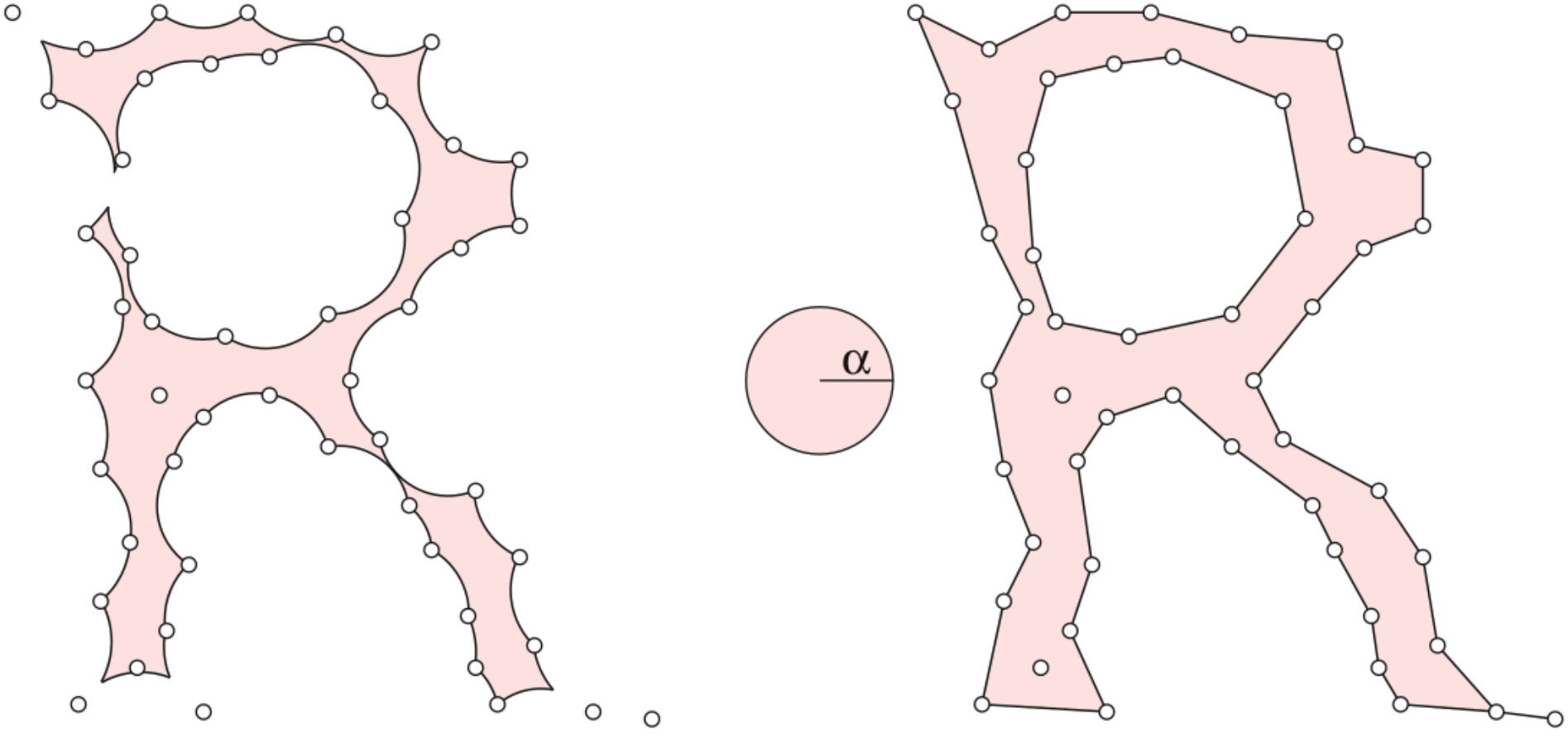
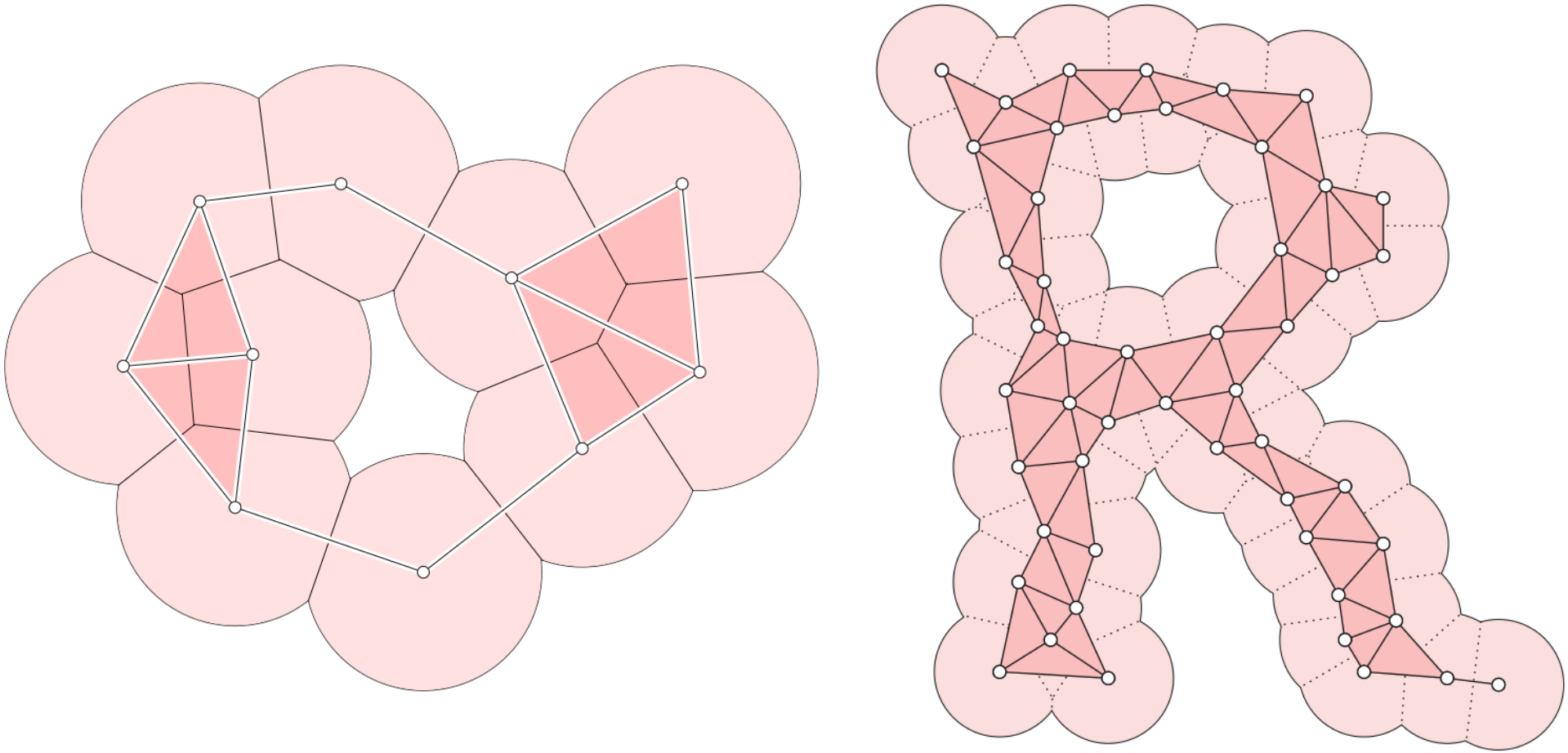
Or: Topology and its applications

# $\alpha$ shapes



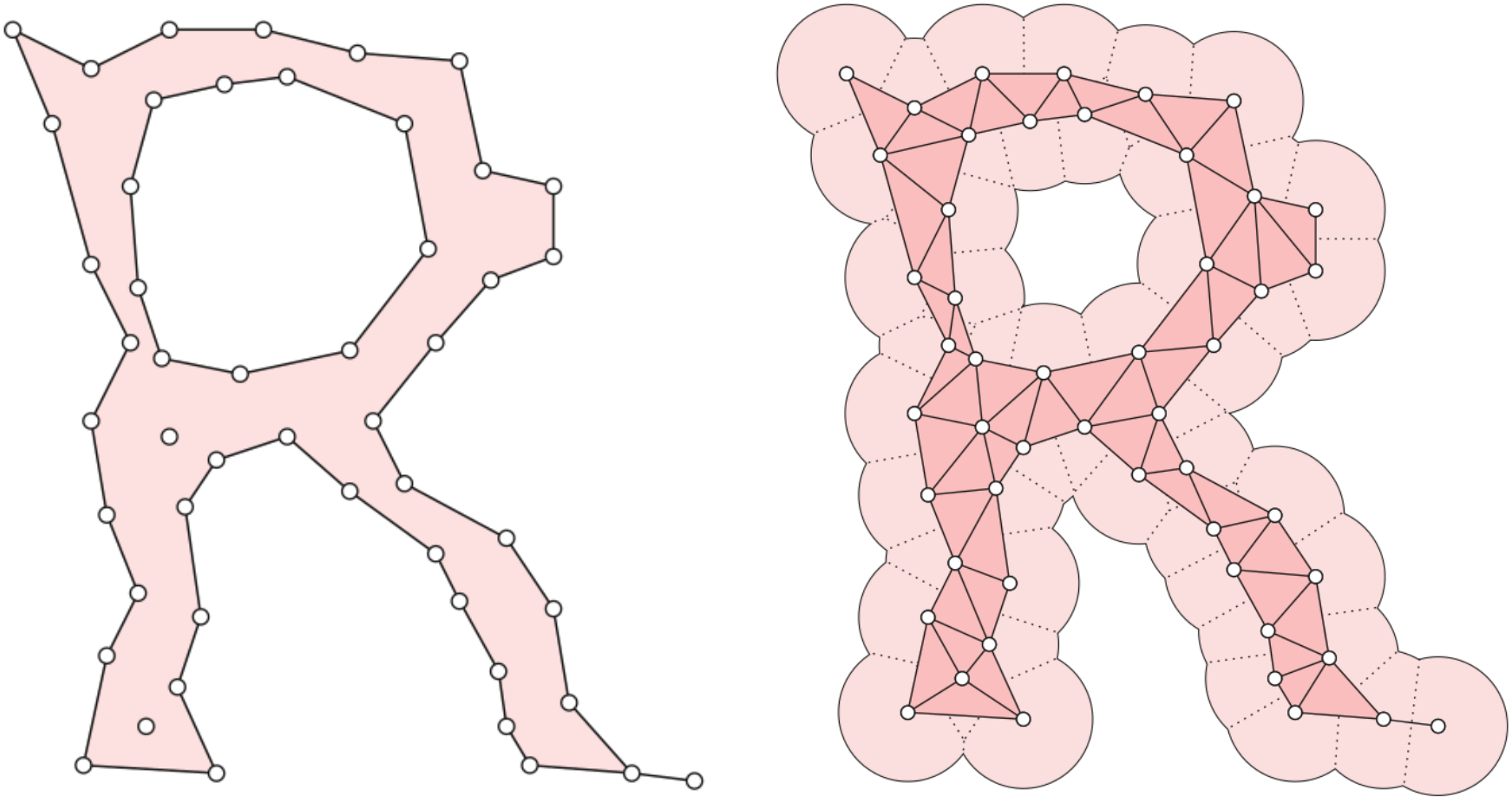

- Say we have a point cloud of data and we want to known its "shape"
- Form discs of radius $\alpha$; the $\alpha$ hull is the complement of the union of the discs hitting no point
- The $\alpha$ shape is obtained by drawing the curved edges straight

# $\alpha$ complexes



- Say we have a point cloud of data and we want to known its "shape"
- Form discs of radius $\alpha$ around the points + connect the points along intersections
- Fill in potential regions and get the $\alpha$ complex for the points

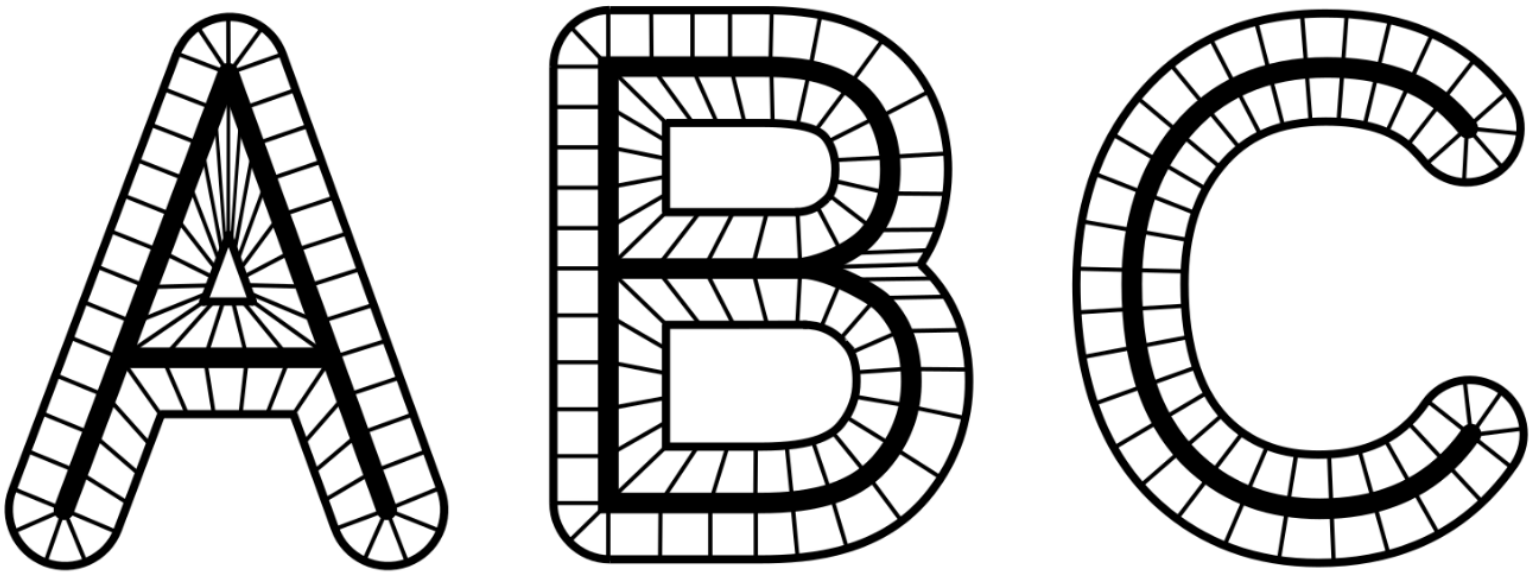
# Hmm, they look kind of similar



- The $\alpha$ shape and complex are not equal
- For example, one construction uses discs not hitting the points, the other uses discs hitting the points
- Question Is there any relation between the two?

# Enter, the theorem

The $\alpha$ shape and complex are homotopy equivalent

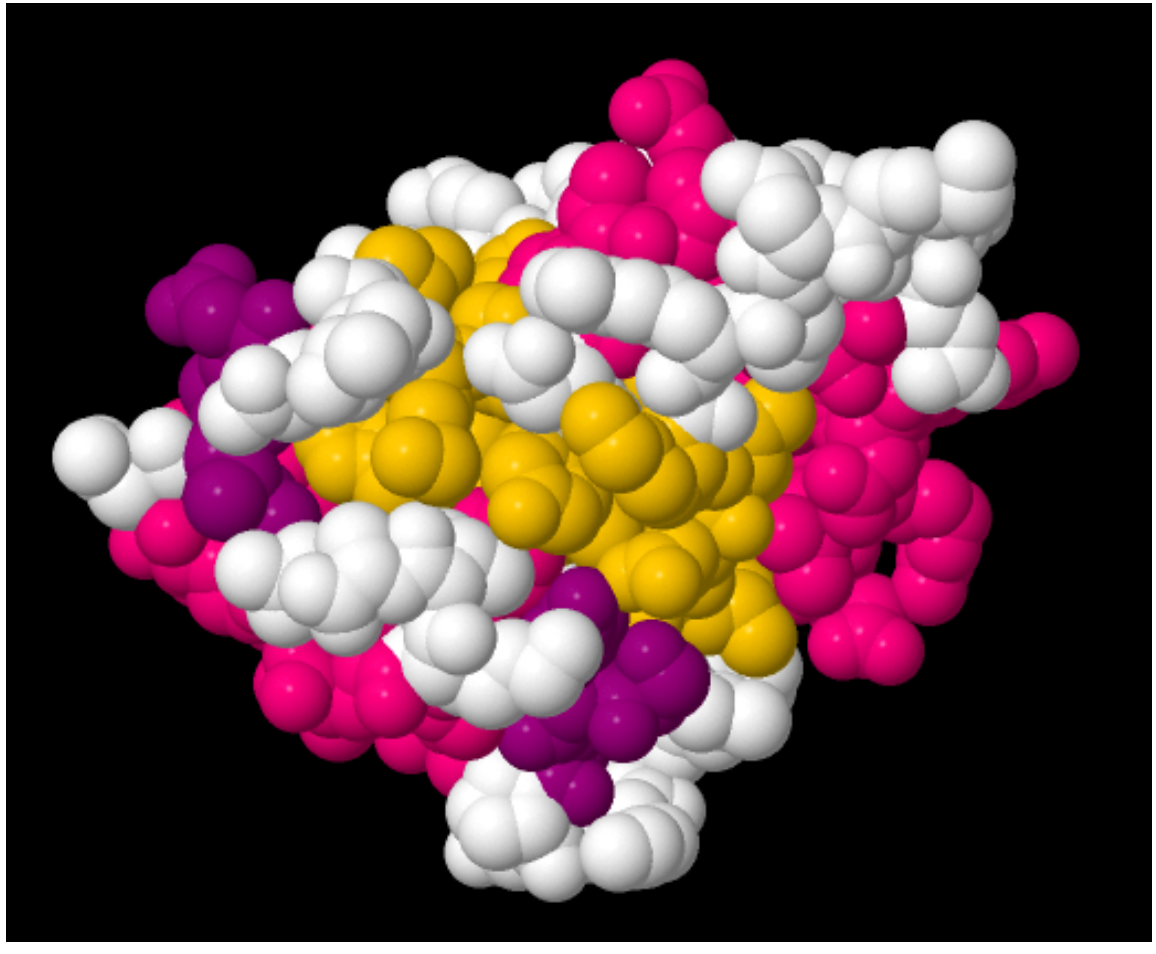


Homotopy types of the graphs underlying the alphabet:

| Genus 0 | Genus 1 | Genus 2 |
|---|---|---|
| CEFGHIJKLMNSTUVWXYZ | ADOPQR | B |

- By the way, everything also works in higher dimensions
- The above is rather tricky in higher dimensions

# Modeling the real world



- A major application of $\alpha$ complexes are molecules modeled as unions of balls
- Example van der Waals diagrams of proteins

**Thank you for your attention!**

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I hope that was of some help.