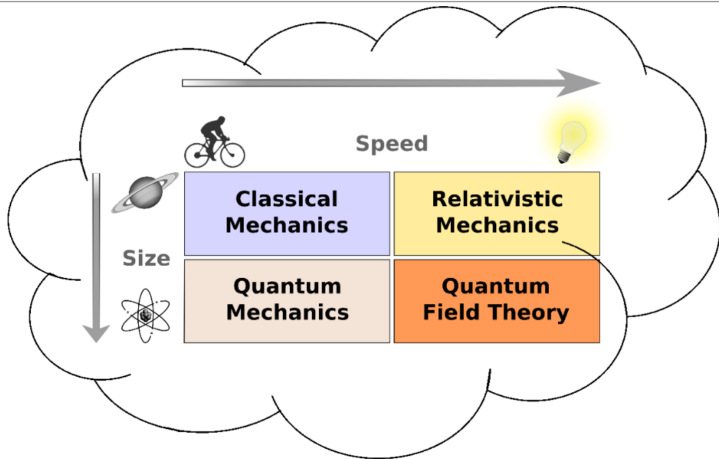**What are...Frobenius algebras and 2d TQFTs?**

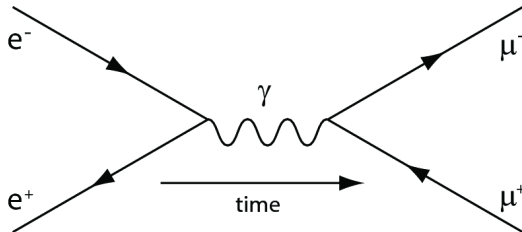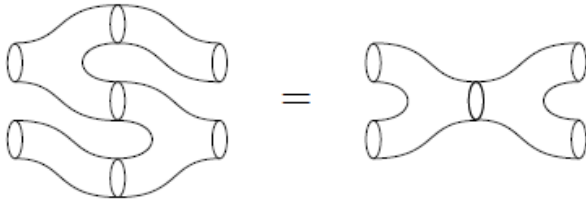Or: Physics and math walk hand in hand

▶ Quantum field theory (QFT) merges relativity and quantum mechanics

▶ Many principles in QFT are explained, or get further insight, in string theory

▶ Both run in parallel have led to fruitful interplay between physics and math

Simple = good !?

- A topological QFT (TQFT) is a toy model within QFT and string theory

- Quote Segal "not realistic as a description of space-time"

- Even this "simplified physics" gives fabulous mathematics
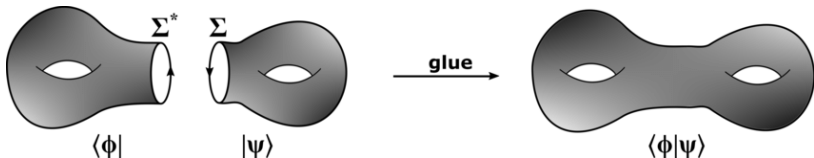
Atiyah–Segal axioms – 2d only and simplified

$\varepsilon : \begin{cases} v_+ \to 0 \quad v_- \to 0 \\ 1_A \to 1_K \end{cases}$

$m : \begin{cases} v_+ \otimes v_+ \to v_+ \quad v_- \otimes v_+ \to v_- \\ v_+ \otimes v_- \to v_- \quad v_- \otimes v_- \to 0 \end{cases}$

$\Delta : \begin{cases} v_+ \to v_+ \otimes v_- + v_- \otimes v_+ \\ v_- \to v_- \otimes v_- \end{cases}$

$u : \quad 1_K \to 1_A$
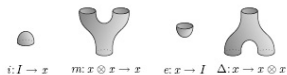
$\langle\phi|$ $\qquad$ Σ^* $\quad$ Σ $\qquad$ **glue** $\qquad$ $\langle\phi|\psi\rangle$

$|\psi\rangle$

► 2d TQFT = assignment circle ↦ vector space, cobordism ↦ linear map

► This is done in such a way that gluing and cutting works locally

► Question How to find 2d TQFTs?

2d TQFTs are the same as commutative Frobenius algebras

$i: I \to x \quad m: x \otimes x \to x \quad e: x \to I \quad \Delta: x \to x \otimes x$
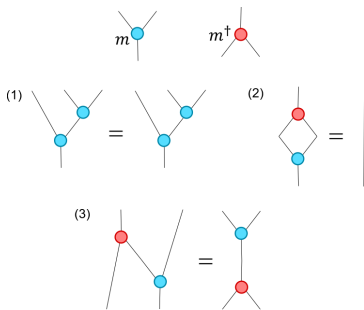
Monoid laws:

Comonoid laws:

Frobenius laws: Commutativity:

▶ Frobenius algebras = objects appearing in classical algebra (next slide)

▶ The point We thus have a link between physics, topology and algebra

Its quite the Venn diagram

m ⬤ $m^\dagger$ ⬤

(1) $\quad = \quad$

(2) $\quad = \quad$

(3) $\quad = \quad$

▶ Frobenius algebras were invented to study **relations** between projective and injective modules ⟿ **classical algebra**

▶ **Fabulous interaction** Classical questions in algebra, ∼50 years later, turned out to be at the heart of **physics and topology**

Thank you for your attention!

I hope that was of some help.