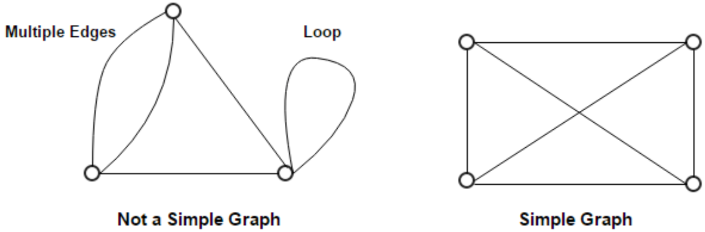
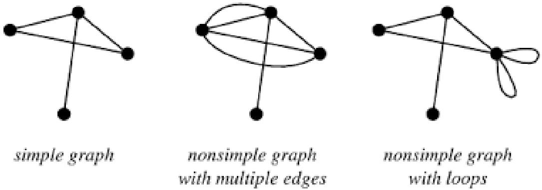
What is...Fagin's theorem?

Or: Almost always true, almost always false

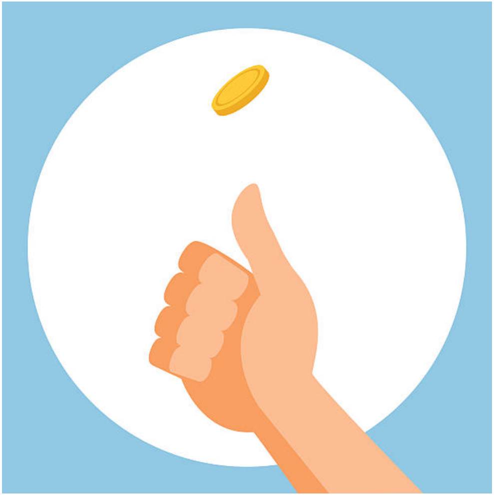
Simple graphs and their random friends





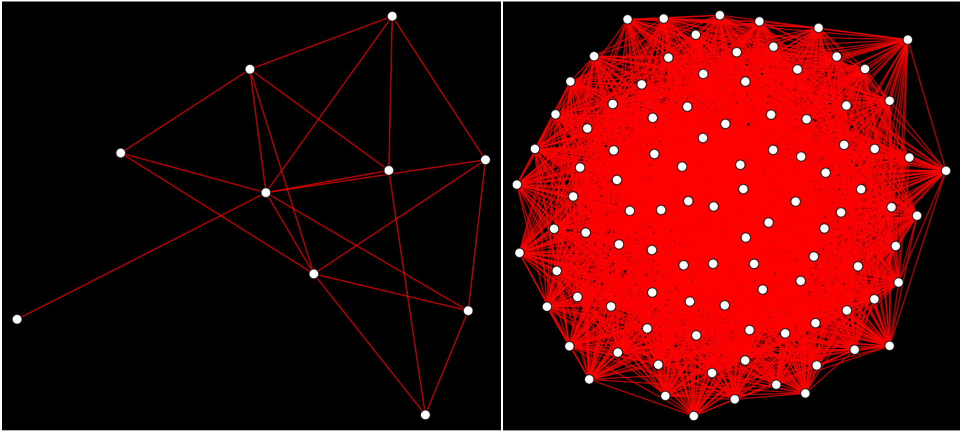
- Simple graph = a graph without multiple edges or loops
- Random (simple) graph = for each pair v, w of vertices with $v \neq w$ put an edge with probability $0 \leq p \leq 1$

Coin toss graphs



- Random graphs in this video are coin toss graphs
- This just means that $p = 0.5$

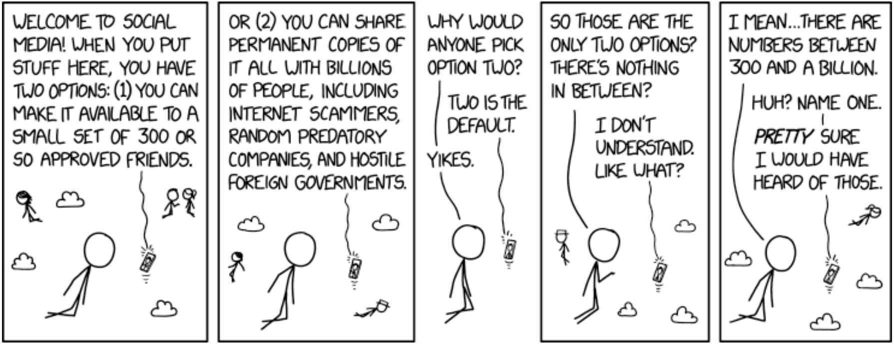
Coin toss graphs with many vertices



- Random graphs are best studied for $n \gg 0$ (n=# vertices)
- Some patterns seem to stabilize
- Note that almost all graphs are very large, *i.e.* $n \gg 0$

Enter, the theorem

A property of first-order logic (fol) holds true either for almost all coin toss graphs or is false for almost all of them Only two options



- I comment on fol graph theory on the next slide
- A bit disappointing : fol graph theory is quite uninteresting

Fol graph theory

$$\exists \forall$$

- Fol graph theory '=' the only adjacency relation and equality, and quantification is only permitted over elements of the graph
- Not fol : "G is connected", "G is bipartite", "G is Hamiltonian", ...
- Example "G is connected" needs quantifying over paths

Thank you for your attention!

I hope that was of some help.