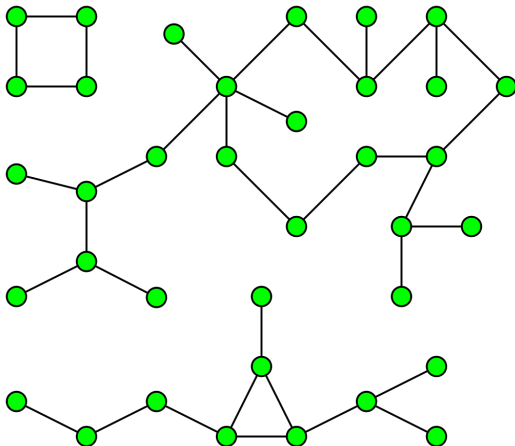


What is...Stone's duality?

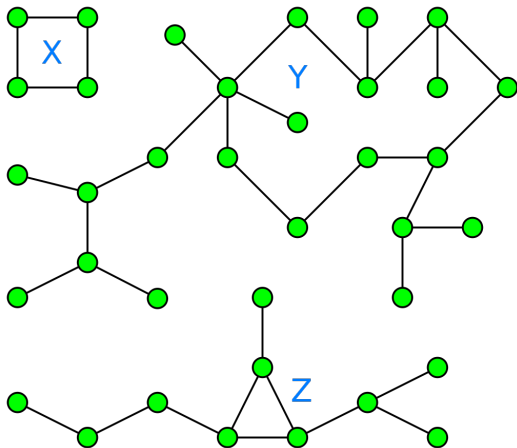
Or: Graphs and logic

Graphs in graphs



- ▶ Consider a large parent graph G
- ▶ Let $B = B(G)$ be the set of all clopen (closed+open) parts of G
- ▶ B is generated by the connected components

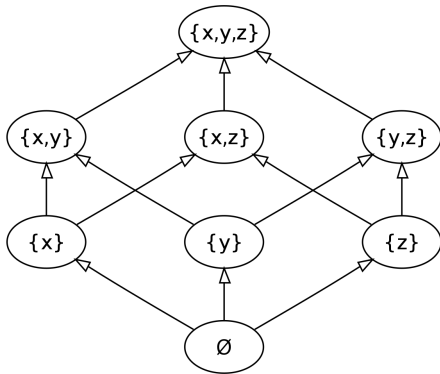
Algebra on graphs



$$B = \{\emptyset, X, Y, Z, X \vee Y, X \vee Z, Y \vee Z, G\}, |B| = 2^3$$

- ▶ B has three operations: \vee =union, \wedge =intersection and \neg =complement
- ▶ Examples $\neg(X \vee Z) = Y$, $(X \vee Z) \wedge (Y \vee Z) = Z$

A Boolean algebra



► $(B, \vee, \wedge, \neg, 0 = \emptyset, 1 = G)$ forms a Boolean algebra

► Boolean algebra = algebraic structure mimicking \vee =or, \wedge =and, \neg =not 0 =false, 1 =true

\wedge	0	1
0	0	0
1	0	1

\vee	0	1
0	0	1
1	1	1

a	0	1
$\neg a$	1	0

Enter, the theorem

One way For every Boolean algebra B there exists a Stone space $S(B)$ such that the clopen sets of $S(B)$ form an algebra isomorphic to B

The other way The collection of subsets of any topological space X that are clopen $S(X)$ is a Boolean algebra

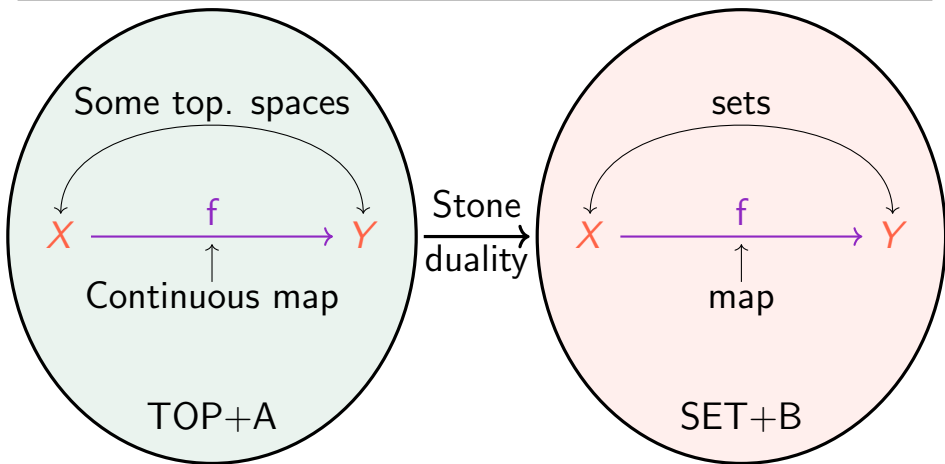
► In other words, there exists a functor

$$\mathbf{Top}^{op} \xrightarrow{S} \mathbf{Bool}$$

The diagram illustrates the mapping S from the opposite category of topological spaces (\mathbf{Top}^{op}) to the category of Boolean algebras (\mathbf{Bool}). It shows a commutative triangle in \mathbf{Top}^{op} with vertices Z , Y , and X . The arrows are $Z \xrightarrow{g^{-1}} Y$, $X \xrightarrow{f^{-1}} Y$, and $X \xrightarrow{g^{-1} \circ f^{-1}} Z$. The mapping S sends this triangle to a commutative triangle in \mathbf{Bool} with vertices $S(Z)$, $S(Y)$, and $S(X)$. The arrows are $S(Z) \xleftarrow{Sg} S(Y)$, $S(X) \xrightarrow{Sf} S(Y)$, and $S(X) \xleftarrow{Sg \circ Sf} S(Z)$.

- Stone space = compact + totally disconnected
- Every Boolean algebra is represented by clopen sets
- Conversely, clopen sets form Boolean algebras

Stone duality



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- ▶ There are **many** categorical dualities between categories of topological spaces and categories of partially ordered sets
 - ▶ These dualities are often collected under **Stone duality**

Thank you for your attention!

I hope that was of some help.