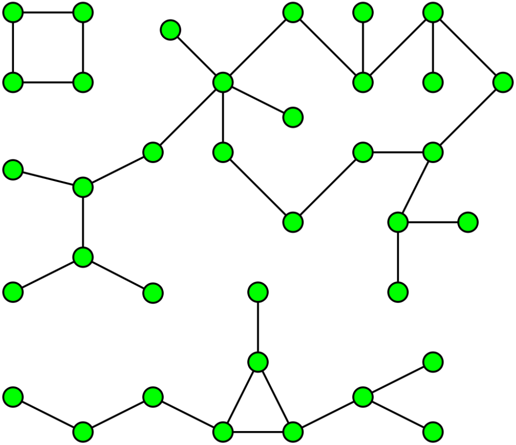
What is...Stone's duality?

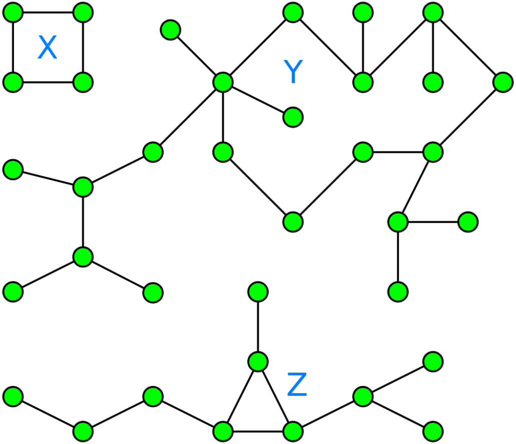
Or: Graphs and logic

Graphs in graphs



- Consider a large parent graph G
- Let $B = B(G)$ be the set of all clopen (closed+open) parts of G
- B is generated by the connected components

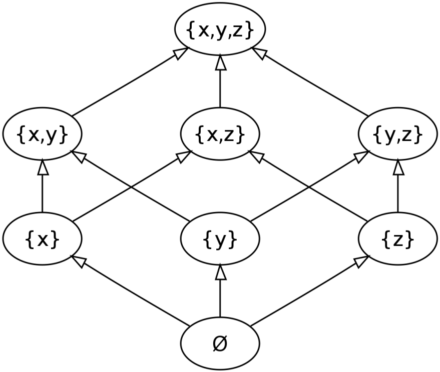
Algebra on graphs



$$B = \{\emptyset, X, Y, Z, X \vee Y, X \vee Z, Y \vee Z, G\}, |B| = 2^3$$

- B has three operations: $\vee$=union, $\wedge$=intersection and $\neg$=complement
- Examples $\neg(X \vee Z) = Y$, $(X \vee Z) \wedge (Y \vee Z) = Z$

A Boolean algebra



- $(B, \vee, \wedge, \neg, 0 = \emptyset, 1 = G)$ forms a Boolean algebra
- Boolean algebra = algebraic structure mimicking $\vee$=or, $\wedge$=and, $\neg$=not 0=false, 1=true

∧	0	1
0	0	0
1	0	1

∨	0	1
0	0	1
1	1	1

a	0	1
¬*a*	1	0

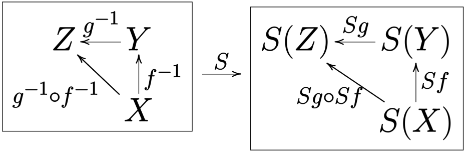
Enter, the theorem

One way For every Boolean algebra B there exists a Stone space $S(B)$ such that the clopen sets of $S(B)$ form an algebra isomorphic to B

The other way The collection of subsets of any topological space X that are clopen $S(X)$ is a Boolean algebra

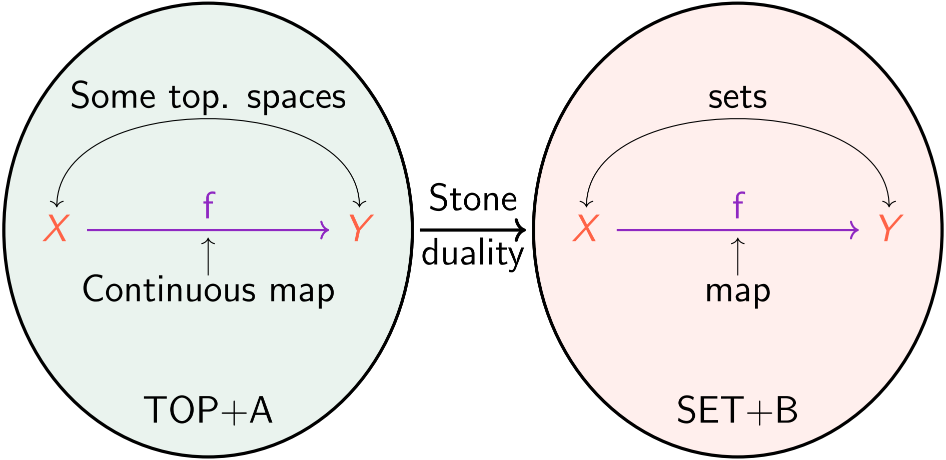
- In other words, there exists a functor

$$\mathbf{Top}^{op} \xrightarrow{S} \mathbf{Bool}$$



- Stone space = compact + totally disconnected
- Every Boolean algebra is represented by clopen sets
- Conversely, clopen sets form Boolean algebras

Stone duality



- There are many categorical dualities between categories of topological spaces and categories of partially ordered sets
- These dualities are often collected under Stone duality

Thank you for your attention!

I hope that was of some help.