

What is...the Garden of Eden theorem?

Or: Ignored by life?

Conway's game of life = "life"

Rules

For a space that is populated:

Each cell with one or no neighbors dies, as if by solitude.

Each cell with four or more neighbors dies, as if by overpopulation.

Each cell with two or three neighbors survives.

For a space that is empty or unpopulated:









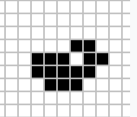

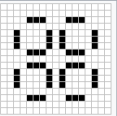


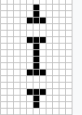
Each cell with three neighbors becomes populated.

Examples



- ▶ **Idea** Easy local rules give complicated global behavior
- ▶ "Life" is a special case of a **cellular automata** and "quite random"
- ▶ **Question** Do all configurations occur?

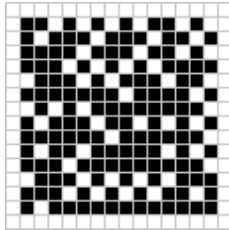
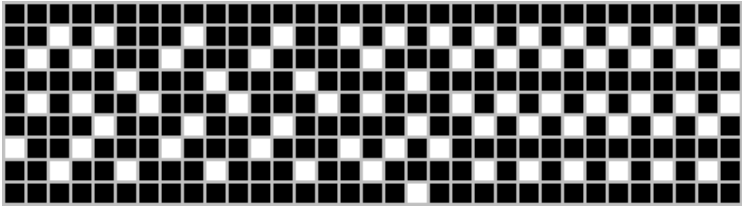
Some first patterns

Still lifes		Oscillators		Spaceships	
Block		Blinker (period 2)		Glider	
Beehive		Toad (period 2)		Light-weight spaceship (LWSS)	
Loaf		Beacon (period 2)		Middle-weight spaceship (MWSS)	
Boat		Pulsar (period 3)		Heavy-weight spaceship (HWSS)	
Tub		Pentadecathlon (period 15)			

► Some patterns **easy** to observe

► **'Problem'** Is there anything we can say about larger patterns?

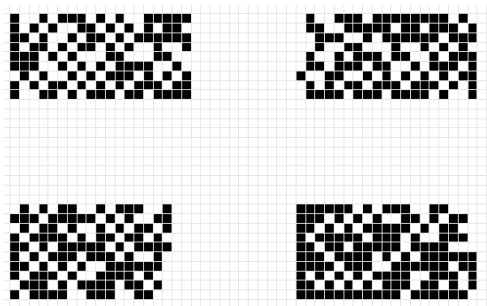
Garden of Eden



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- ▶ Garden of Eden = a configuration that has no predecessor (“created out of nowhere”)
 - ▶ It can be the initial configuration but cannot arise in any other way
 - ▶ Question Do these exist?

Enter, the theorem

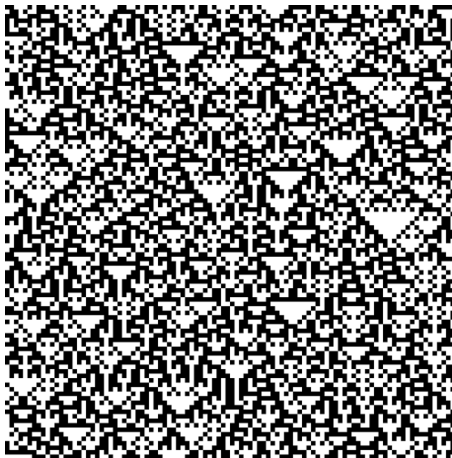
A (euclidean) cellular automaton has a Garden of Eden **if and only** if it has twins



This **applies** Conway's game of life and Gardens of Eden exist

- ▶ Twins = two patterns that can be exchanged without changing the future configurations
- ▶ For one-dimensional cellular automata, Gardens of Eden can be found by an efficient algorithm, but for higher dimensions this is an undecidable problem

Rule 90 is very inclusive



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- ▶ Not all cellular automata have Gardens of Eden
 - ▶ Example The famous Rule 90 has no Gardens of Eden

Thank you for your attention!

I hope that was of some help.