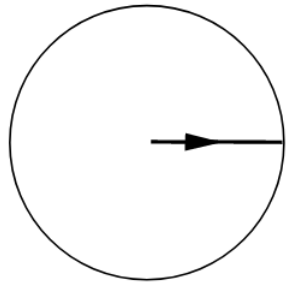
What is...Dirac's belt trick?

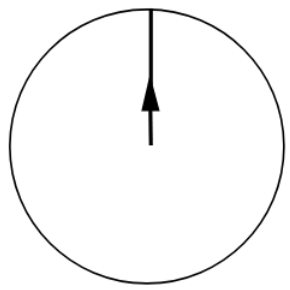
Or: 720° is it!

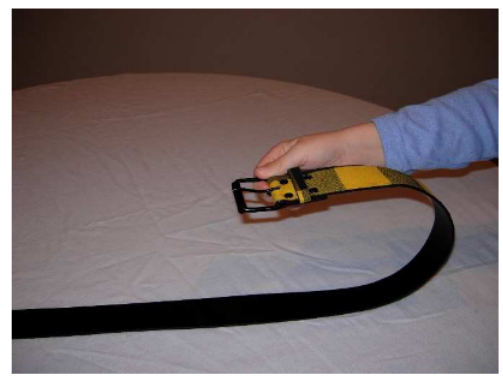
The trick

- Rotate the belt by twice 360° around a vertical axis
- The belt tangles up and looks like it is in a nontrivial state
- The belt can be untangled without any further rotation

Rotations in three-space by 180°

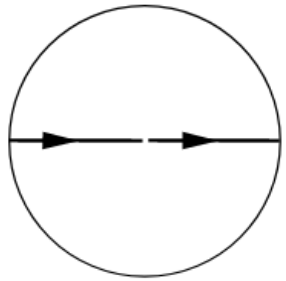


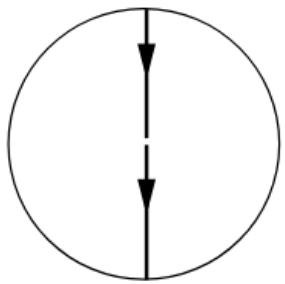


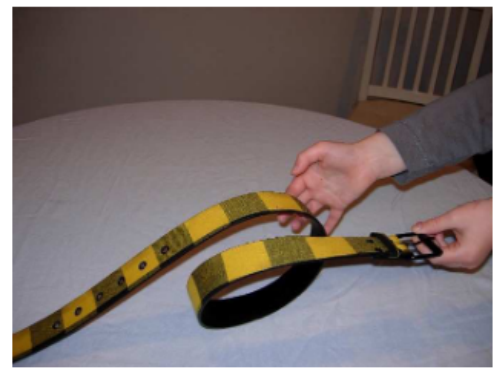


- Top The belt is twisted by 180° about the axis parallel to the length of the belt
- Bottom The belt is rotated by 180° about an axis in the plane of the table perpendicular to the length of the belt

Rotations in three-space by 360°





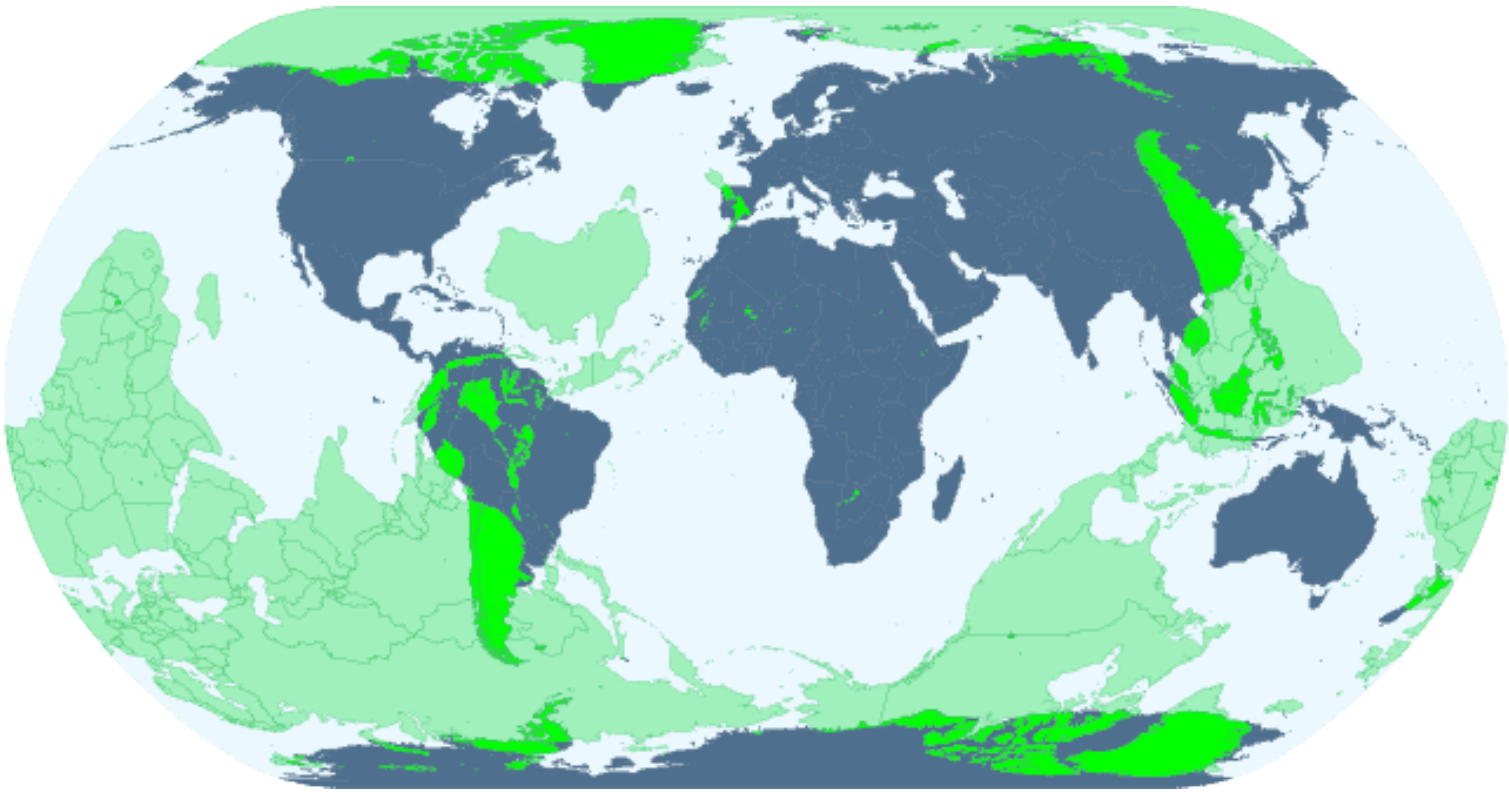


- Top The belt is twisted by 360° about the axis parallel to the length of the belt
- Bottom The belt is rotated by 360° about an axis in the plane of the table perpendicular to the length of the belt

Enter, the theorem

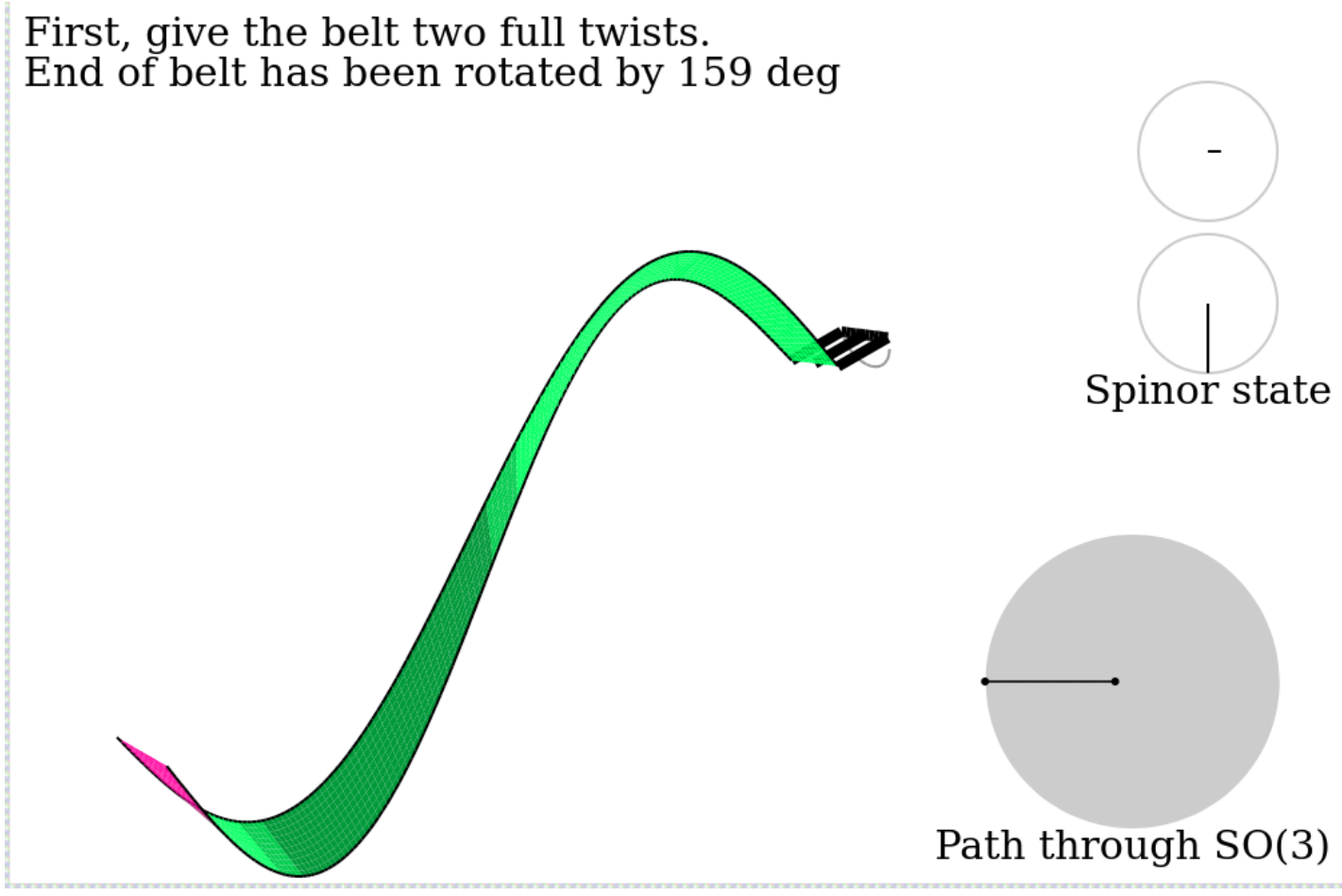
$SO_3(\mathbb{R})$ is not simply connected and its π_1 is $\mathbb{Z}/2\mathbb{Z}$

- $SO_3(\mathbb{R})$ = rotation group on $\mathbb{R}^3$
- Topologically $SO_3(\mathbb{R}) = S^3$/antipodal points



- Belt trick = a loop by 360° is nontrivial, doing it twice is trivial

360° and 720° are loops



- We draw $SO_3(\mathbb{R})$ as a sphere
- Recall that antipodal points are identified
- Then the rotation by 360° loops around once and 720° loops around twice

Thank you for your attention!

I hope that was of some help.