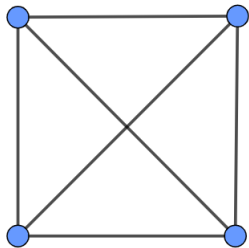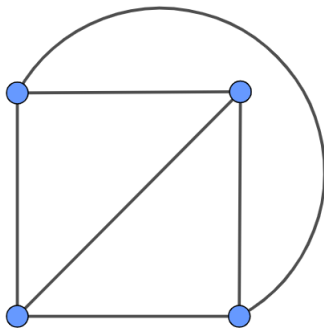**What is...the genus of a graph?**

Or: Building bridges

Abstract versus embedded

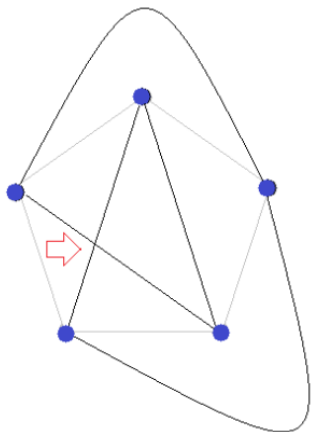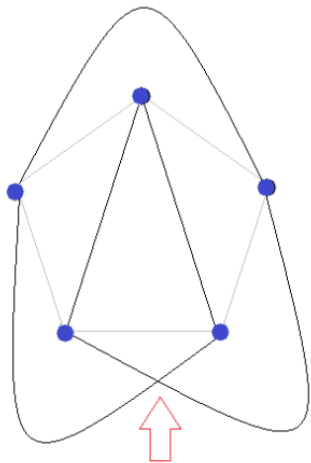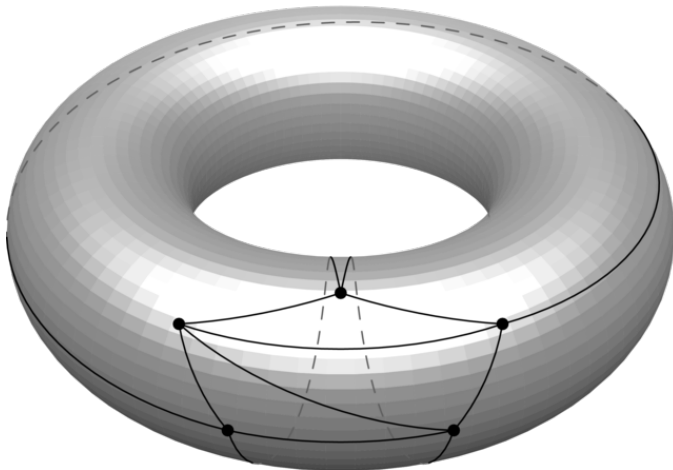Planar graph
(K_4)

Planar embedding
of K_4

- ▶ A graph is an abstract object

- ▶ We can think of embedded graphs that are realized somewhere

- ▶ For example, a planar graph embeds into the plane

Not all graphs are planar

- Some graphs are not planar

- For example, the complete graph K_5 is not planar

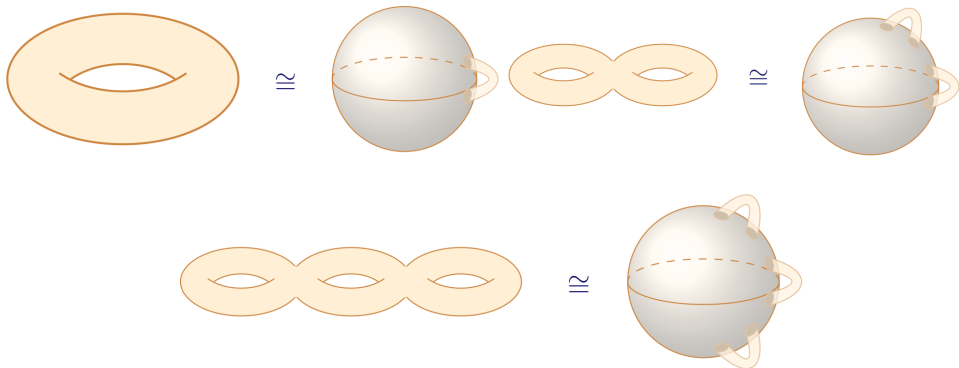- Question Can graphs always be embedded in a 2d object?

K_5 on the torus

- K_5 does not embed into the plane "=" S^2
- K_5 does embed into the torus
- Question Is there something we can say in general?

Every graph can be embedded into some orientable surface
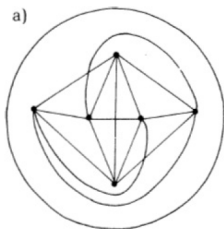The minimal number of handles needed is the genus of the graph

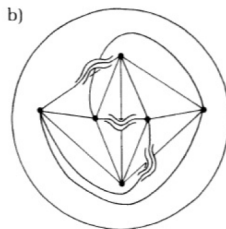▶ Genus is a notion from topology and equals the number of handles

Every closed orientable surface admits such a description

▶ Computing the genus of a graph is NP-complete = very hard

Proof? Sure!

a) K_6 on S_0

b) K_6 on S_0-with-three-handles

- ▶ Build a bridge = handle for every crossing

- ▶ Since bridges redirect the path, the result has no crossings

- ▶ The genus is then at most the number of bridges

Thank you for your attention!

I hope that was of some help.