**What is...the infinitude of prime knots?**

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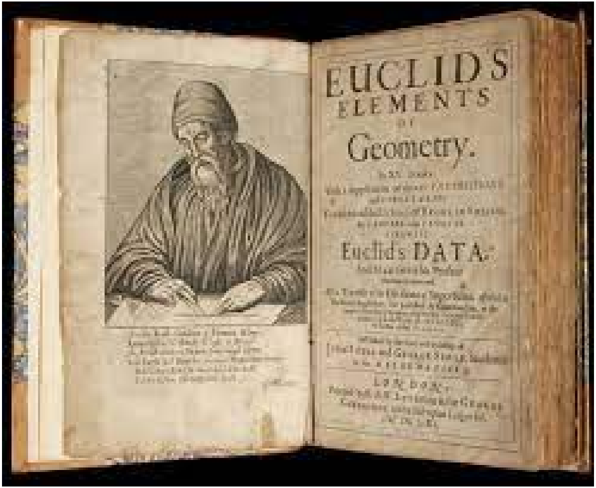
Or: Euclid's theorem for knots

# Euclid's theorem

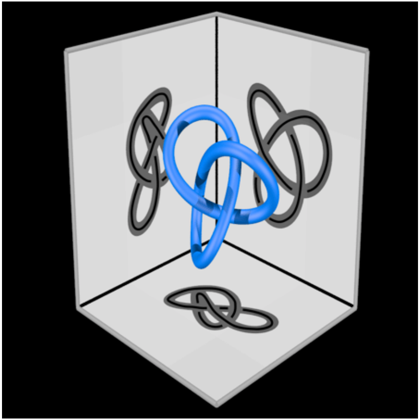
$$(p = ab) \Rightarrow (a = 1 \text{ or } b = 1)$$

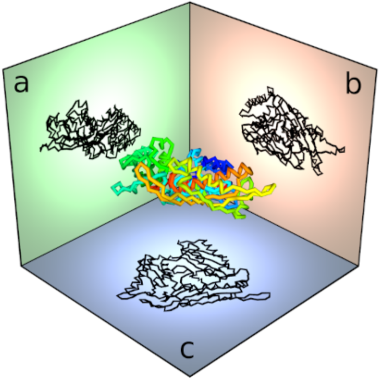
$$\#\text{primes} = \infty$$




- Multiplication is a basic operation of arithmetic
- Primes are the elements of multiplication
- Euclid's theorem ∼300BC There are infinitely many primes

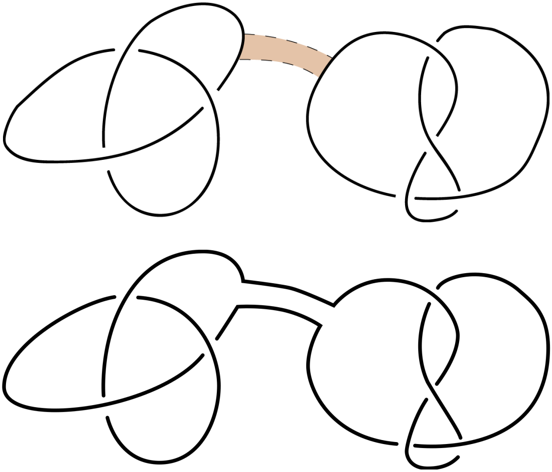
# Knot theory






- A knot is a closed string (a circle $S^1$) in three space
- Knots are often studied by projections to the plane Shadows
- Question Is there a basic operation of knot theory?

## Connected sum #

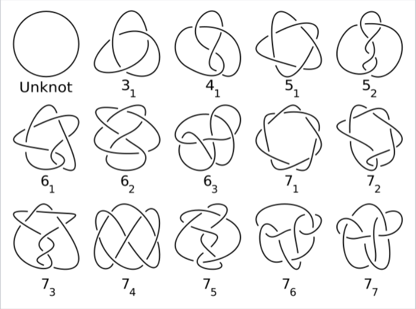


- The connected sum is an operation much like multiplication
- Prime knots: $(K = L\#M) \Rightarrow L$ is trivial or $M$ is trivial
- How many prime knots are there?

## Enter, the theorem

**Euclid's theorem for knots ∼19??** There are infinitely many prime knots

- Actually, there are quite a few prime knots:

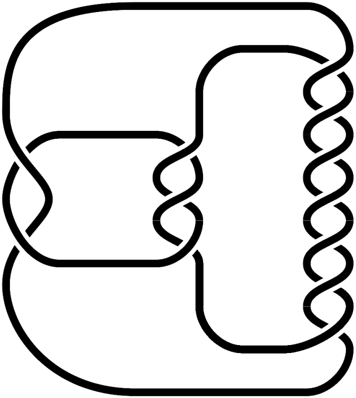



A chart of all prime knots with seven or fewer crossings, not including mirror-images, plus the unknot (which is not considered prime).

- **Proof** The torus knot $T_{2,q}$ for $q > 1$ odd is prime and has genus $(q-1)/2$

## Pretzel primes

$P(-2, 3, 7)$:



- Euclid's theorem has many proofs – and so does its knotty version
- Proof 2 The pretzel knot $P(p, 2p-1, 2p+1)$ for $p > 1$ odd is prime and has a Jones polynomial of min power $A^{-16p}$

**Thank you for your attention!**

I hope that was of some help.