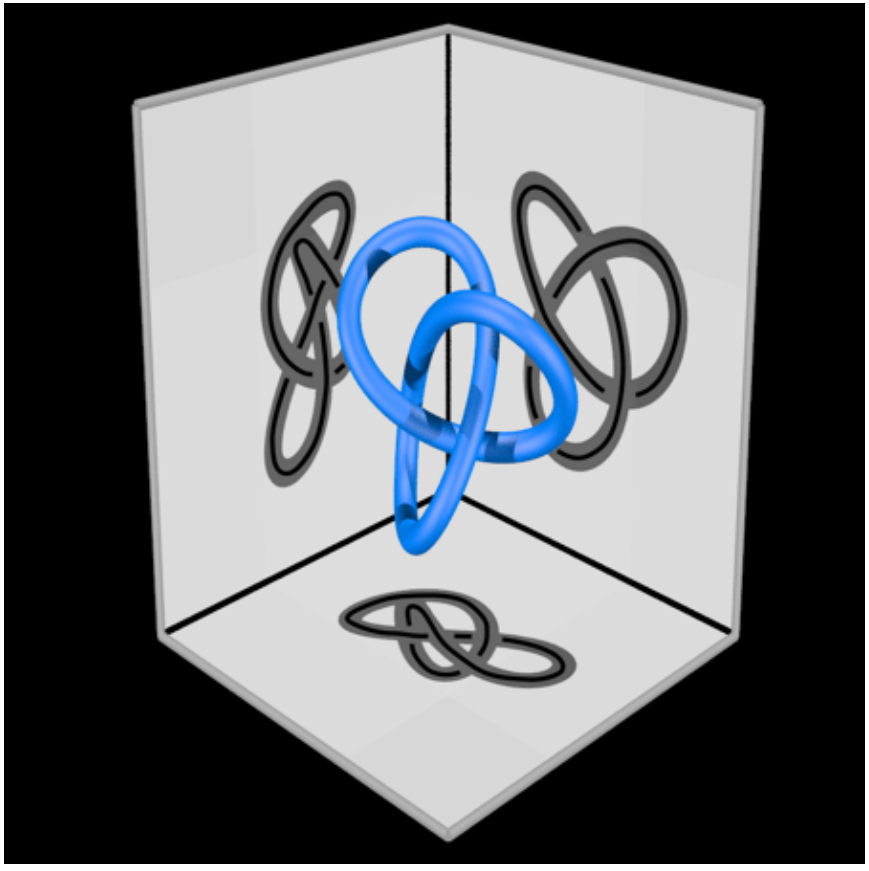
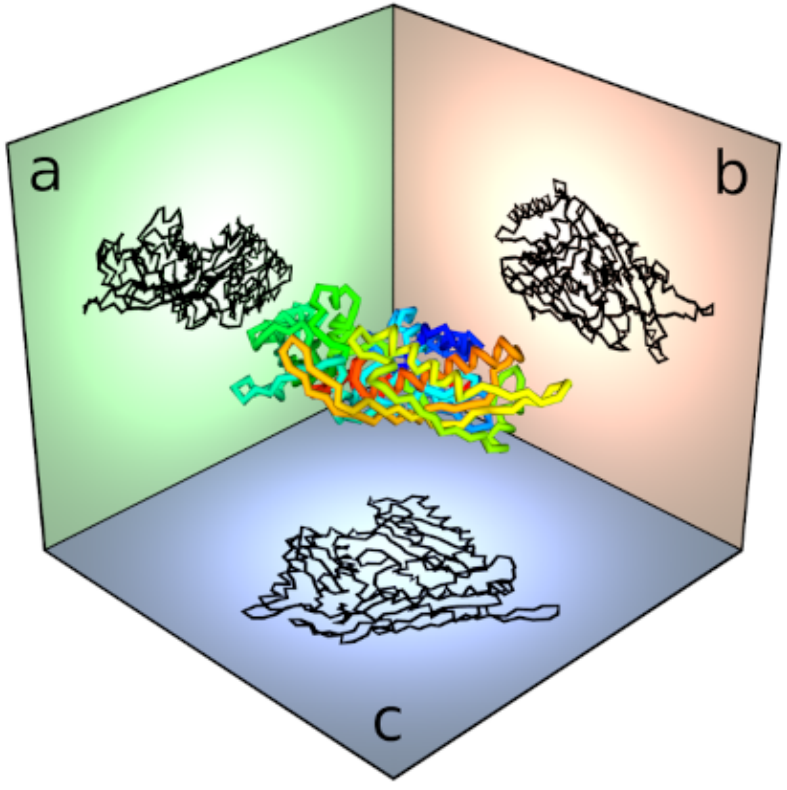
What are...torus knots?

Or: Wind p times, wind q times

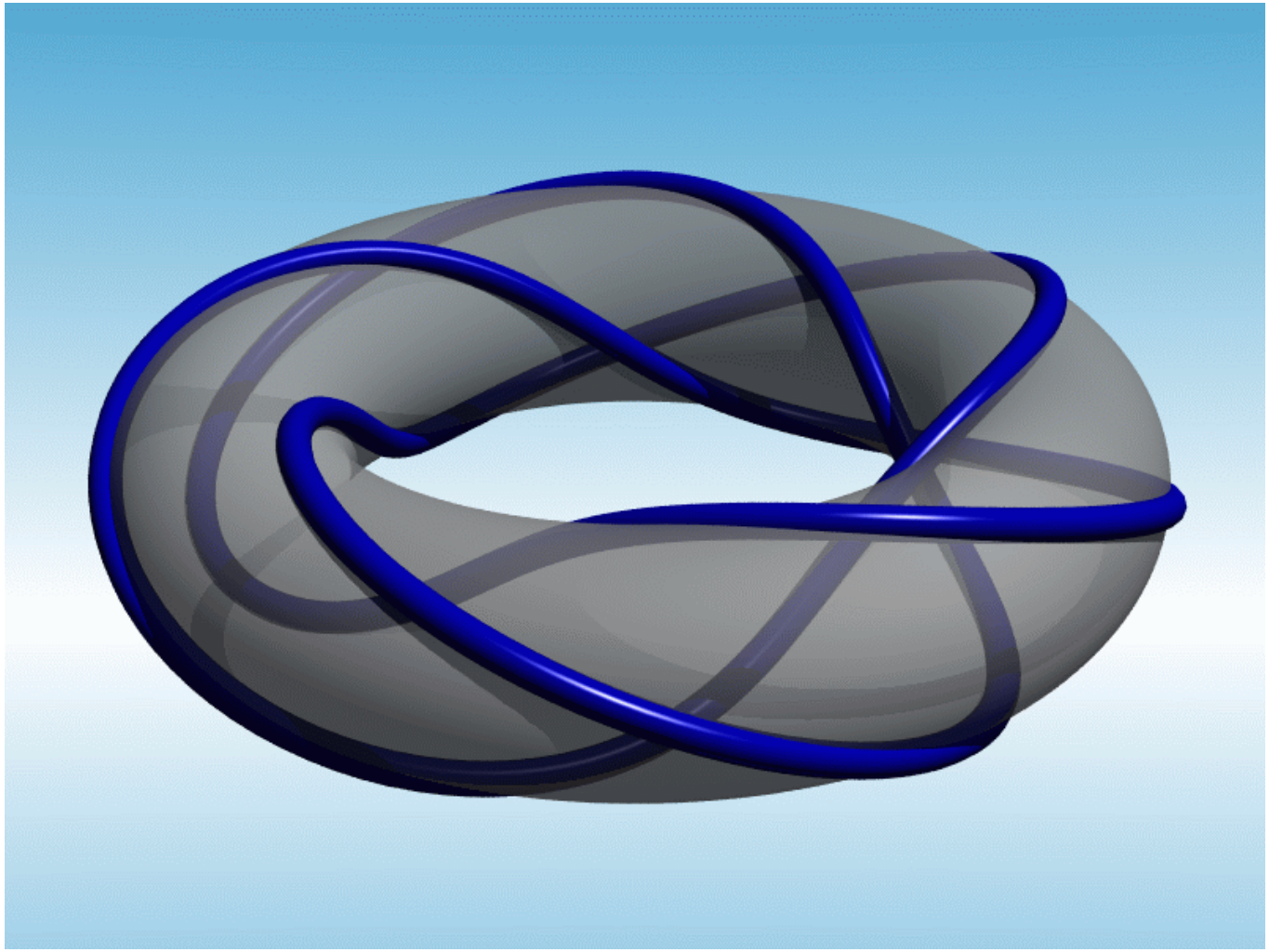
Knots in three space





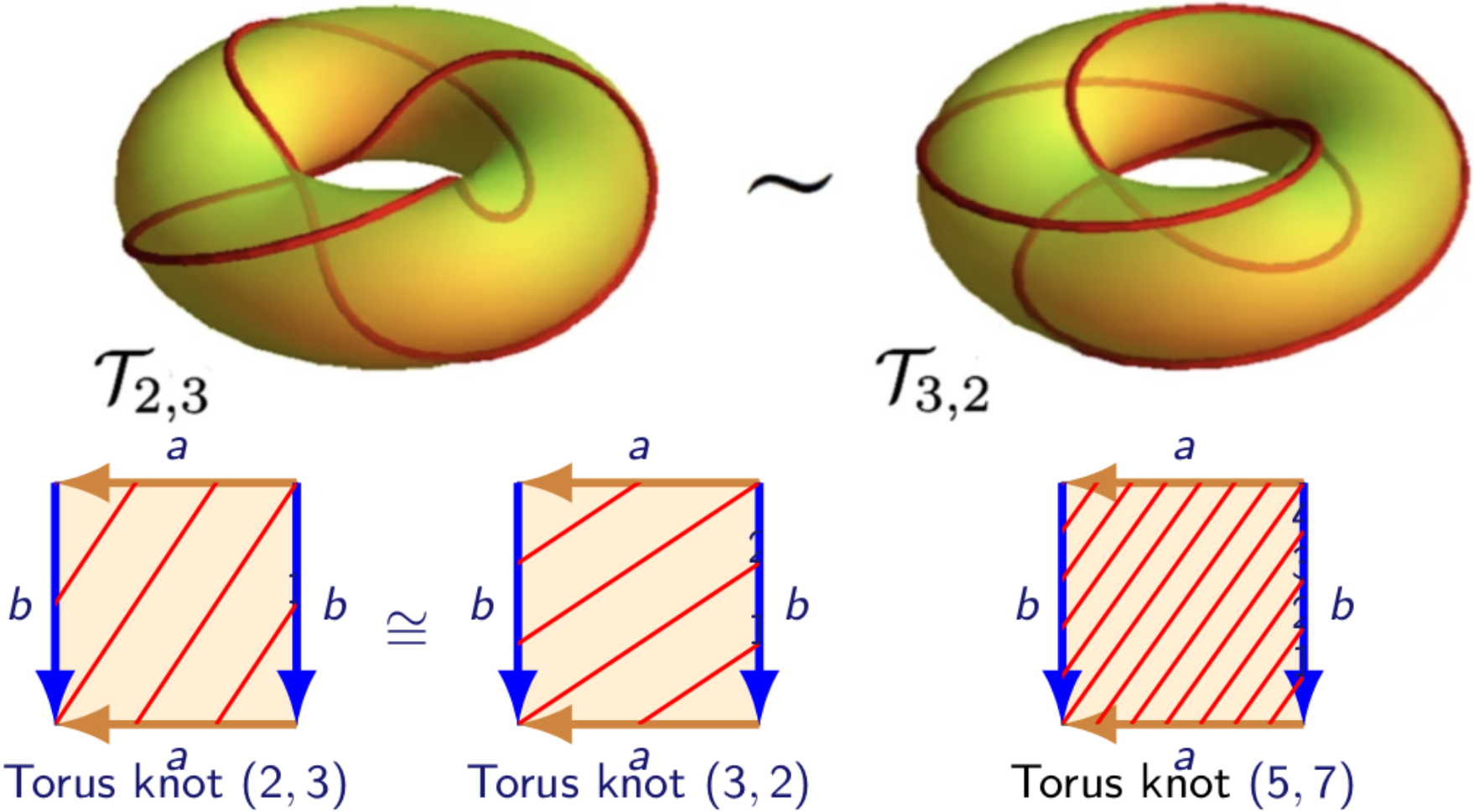
- A knot is a closed string (a circle S^1) in three space
- Knots are often studied by projections to the plane Shadows
- Question Can knots be embedded into some other space?

Knots on a torus



- Take a standard embedded torus T is three space
- A torus knot is a special kind of knot that lies on T without intersections
- Questions Are all knots torus knots? If not, are torus knots "special"?

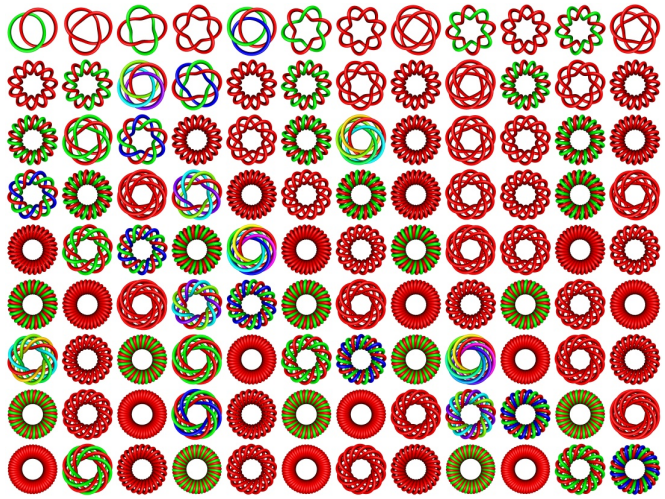
p and q



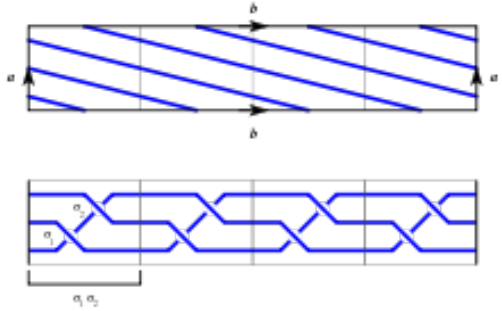
- Fix $p, q \in \mathbb{Z} \setminus \{0\}$ with $gcd(p, q) = 1$
- The (p, q)-torus knot $\mathcal{T}_{p,q}$ is the closed path $\{(x, y) \in T | py \equiv qx\}$ on the standard polygonal decomposition of the torus on the unit square
- The condition $gcd(p, q) = 1$ ensures that the picture closes up

Enter, the theorem

All torus knots are of the form $\mathcal{T}_{p,q}$



- $\mathcal{T}_{p,q}$ is trivial if and only if either p or q is equal to 1 or -1
- There are only very few redundancies, e.g. $\mathcal{T}_{p,q} \cong \mathcal{T}_{q,p}$
- A braid word for $\mathcal{T}_{p,q}$ is $(\beta_1 ... \beta_{p-1})^q$



How many torus knots are there?

A051764 Number of torus knots with n crossings.

0, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 1, 2, 1, 1, 0, 1, 1, 2, 1, 1, 1, 1, 1, 2, 2, 1, 0, 1, 2, 2, 1, 2, 1, 1, 1, 2, 1, 1, 0, 1, 2, 2, 1, 1, 2, 1, 1, 2, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 1, 3, 2, 2, 1, 1, 2, 2, 1, 1, 2, 1, 1, 2, 2, 2, 1, 1, 2, 2, 1, 1, 1, 2, 1, 2, 3, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 3, 1, 1, 1, 1, 3, 3 (list; graph; refs; listen; history; text; internal format)

A002863 Number of prime knots with n crossings.
(Formerly M0851 N0323)

0, 0, 1, 1, 2, 3, 7, 21, 49, 165, 552, 2176, 9988, 46972, 253293, 1388705, 8053393, 48266466, 294130458 (list; graph; refs; listen; history; text; internal format)

A086825 Number of knots (prime or composite) with n crossings.

1, 0, 0, 1, 1, 2, 5, 8, 26 (list; graph; refs; listen; history; text; internal format)

- One can ask how many knots can "run around a torus"
- There are infinitely many nonequivalent torus knots
- However, "almost no" knot is a torus knot

Thank you for your attention!

I hope that was of some help.