What is...the jeu de taquin?
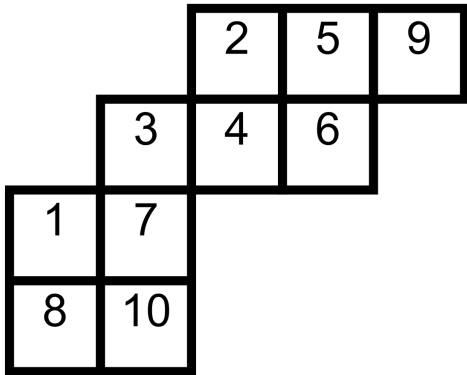
Or: Teasing permutations

Today: the 15 puzzle – kind of

- To solve the puzzle, the numbers must be rearranged into order
- Jeu de taquin (teasing game) is the French name for the 15 puzzle
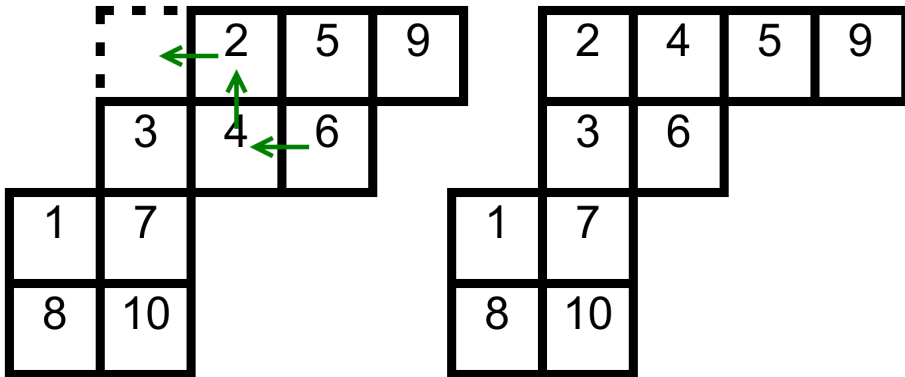- Today A math version of it

Skew things

$(5,4,2,2)/(2,1)$:

- ▶ Skew Young diagram = pair of partitions (λ, μ)
- ▶ Skew Young tableaux = same but with number filling
- ▶ Standard = numbers increase along rows and columns

Jeu de taquin

- Jeu de taquin on skew things "=" the 15 puzzle moves
- Slides must respect the set of skew Young tableaux
- Jeu-de-taquin equivalence = tableaux up to jeu de taquin moves

Enter, the theorem

Row reading + jeu-de-taquin equivalence give a presentation of the plactic monoid

- Plactic monoid = words in $\mathbb{N}$ modulo $yzx = yxz$ whenever $x < y \leq z$ and $xzy = zxy$ whenever $x \leq y < z$
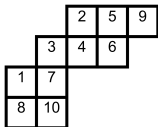- The plactic monoid is modeling permutations so the jeu-de-taquin gives a different way to think about permutations

There is an explicit bijection

Permutations $\rightarrow$ pairs of YT (P, Q)
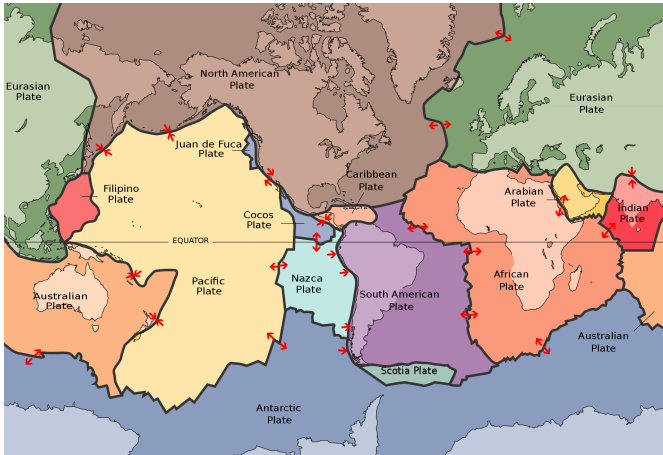
with an explicit inverse

Permutations $\leftarrow$ pairs of YT (P, Q)

- Row reading means

$\leadsto 2, 5, 9, 3, 4, 6, 1, 7, 8, 10$

Monoïde plaxique

▶ Plactic may refer to plate tectonics (tectonique des plaques in French)

▶ The relations involve sliding across each other in analogy to tectonic plates

Thank you for your attention!

I hope that was of some help.