# What is...the dimension of a graph?

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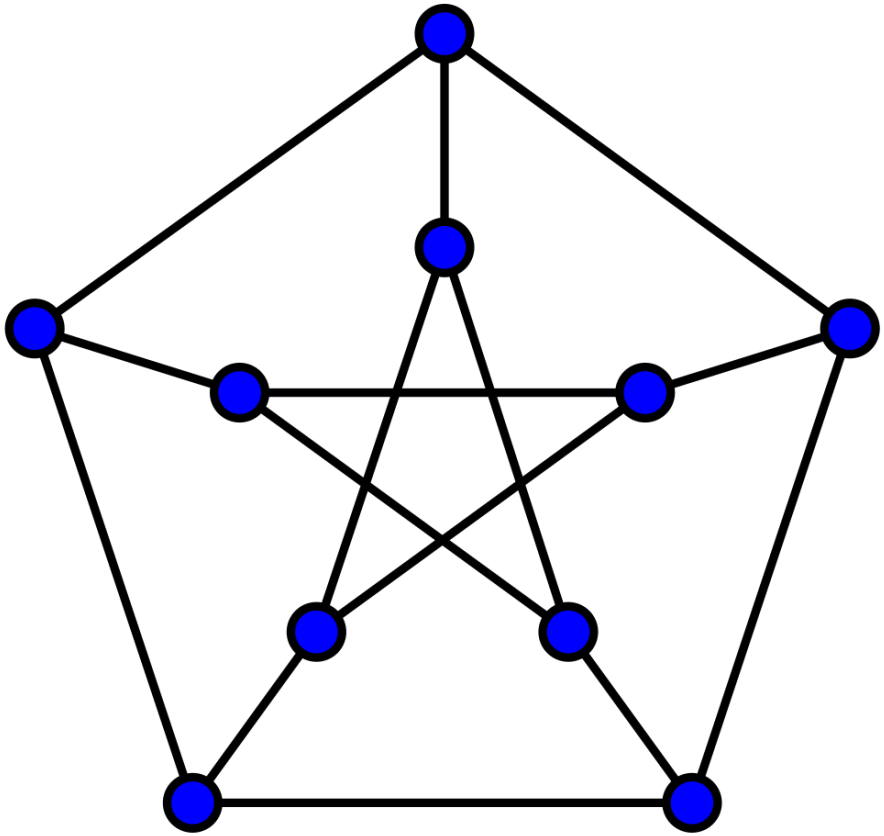
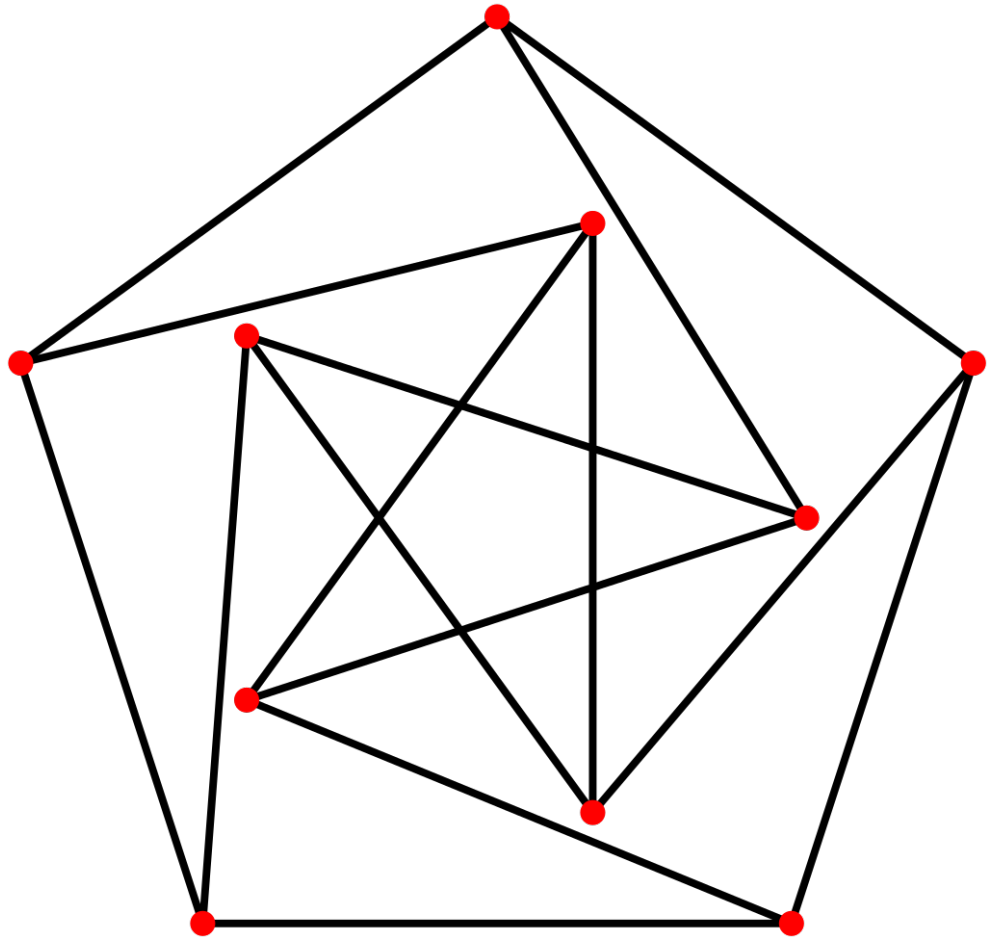
Or: 1d objects can be high dimensional?

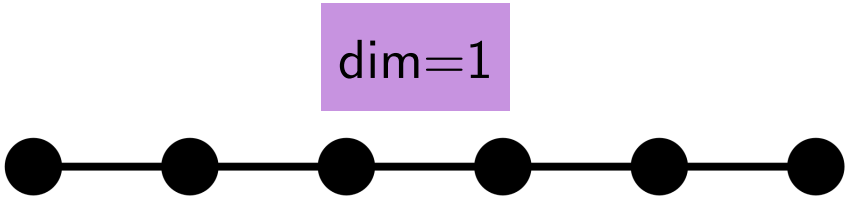
# Graphs are abstract objects, but...





- Graph = abstract collection of vertices and edges Doesn't live anywhere
- But we can ask for nice realizations
- For example, the right realization is better than the left as edges have the same distance

# Low dimensional graphs

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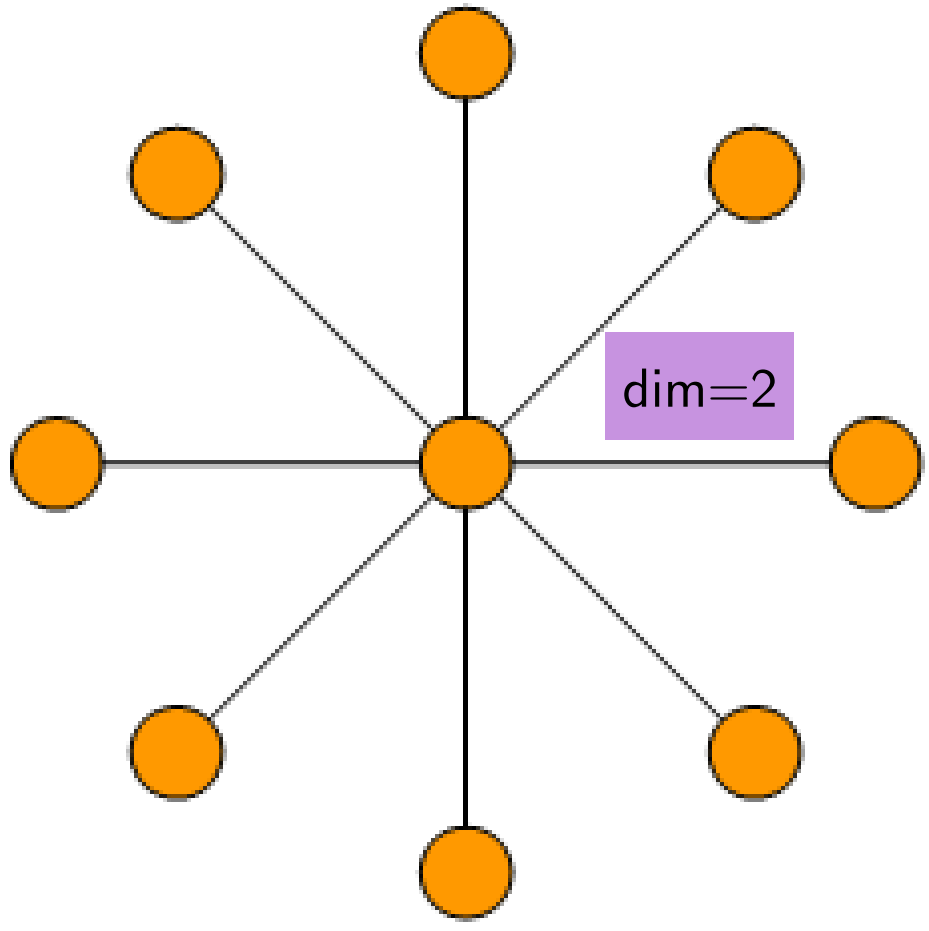
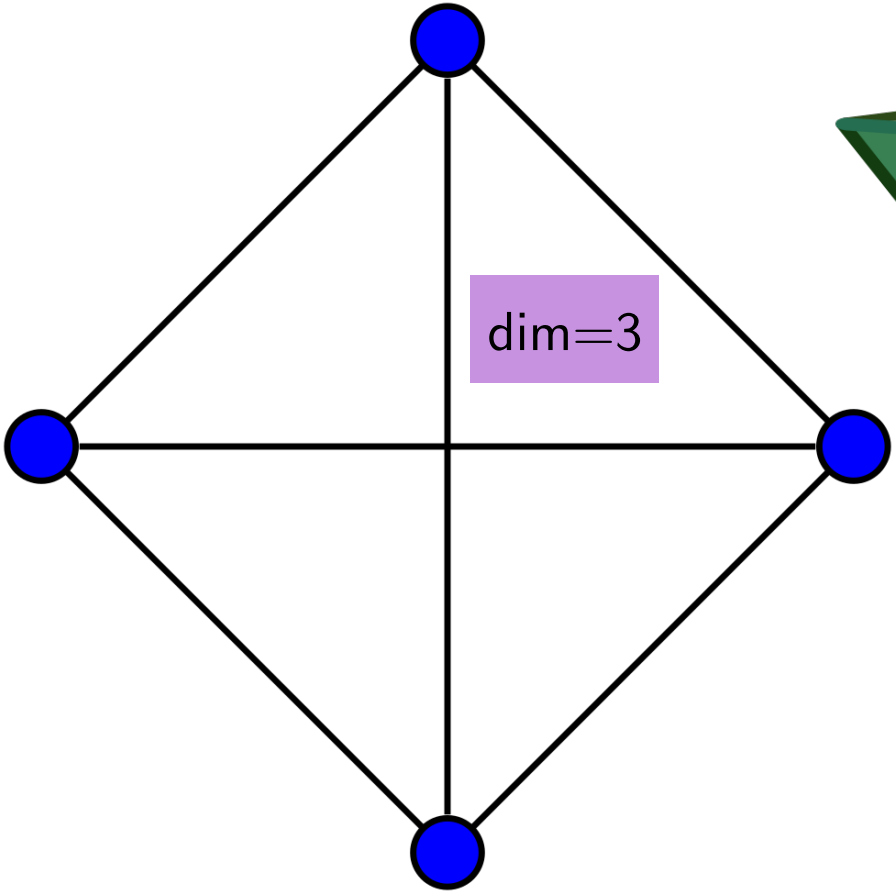



- A graph fits nicely into $\mathbb{R}^n$ if we can draw it in $\mathbb{R}^n$ with all edges of equal length
- $dim(G)$ is the minimum $n$ such that $G$ fits nicely into $\mathbb{R}^n$ Dimension
- Problem Can we say anything about $dim(G)$?

# Higher dimensional graphs

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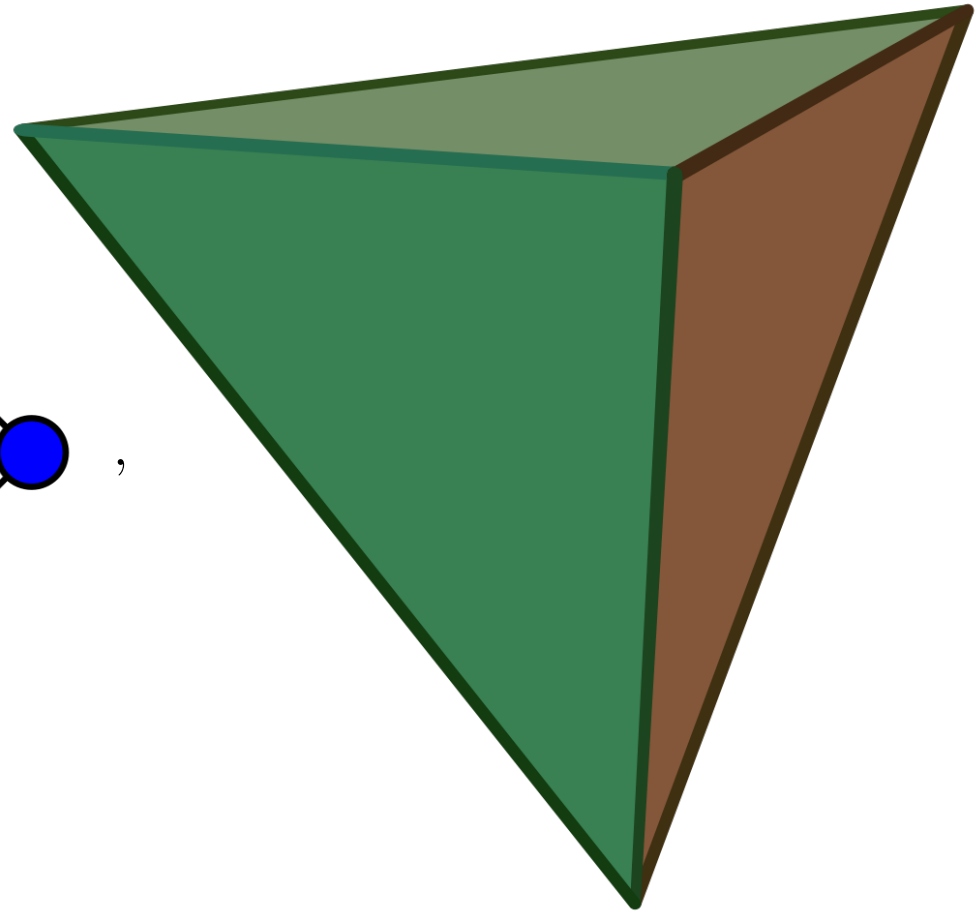


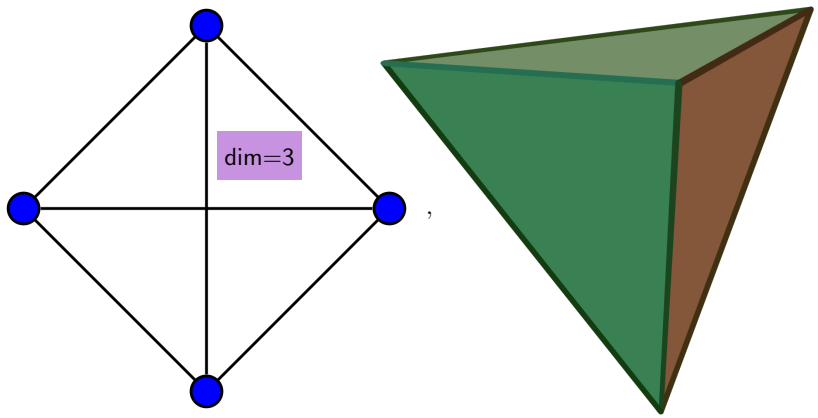
- The tetrahedron graph is nicely realized by the tetrahedron and we cannot do better
- **Problem** Is there any maximal possible value for $dim(G)$?
- **Problem** Can we bound $dim(G)$ using intrinsic properties of $G$?

## Enter, the theorems

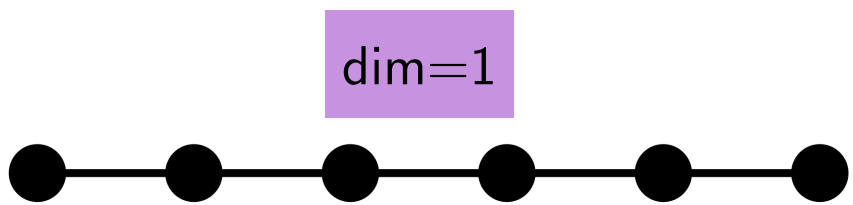
There are graphs of arbitrary dimension as $dim(K_n) = n - 1$ but we always have

$$dim(G) \leq 2 \cdot \mathrm{maxdeg}(G)$$

- We have seen $dim(K_4) = 4 - 1 = 3$:

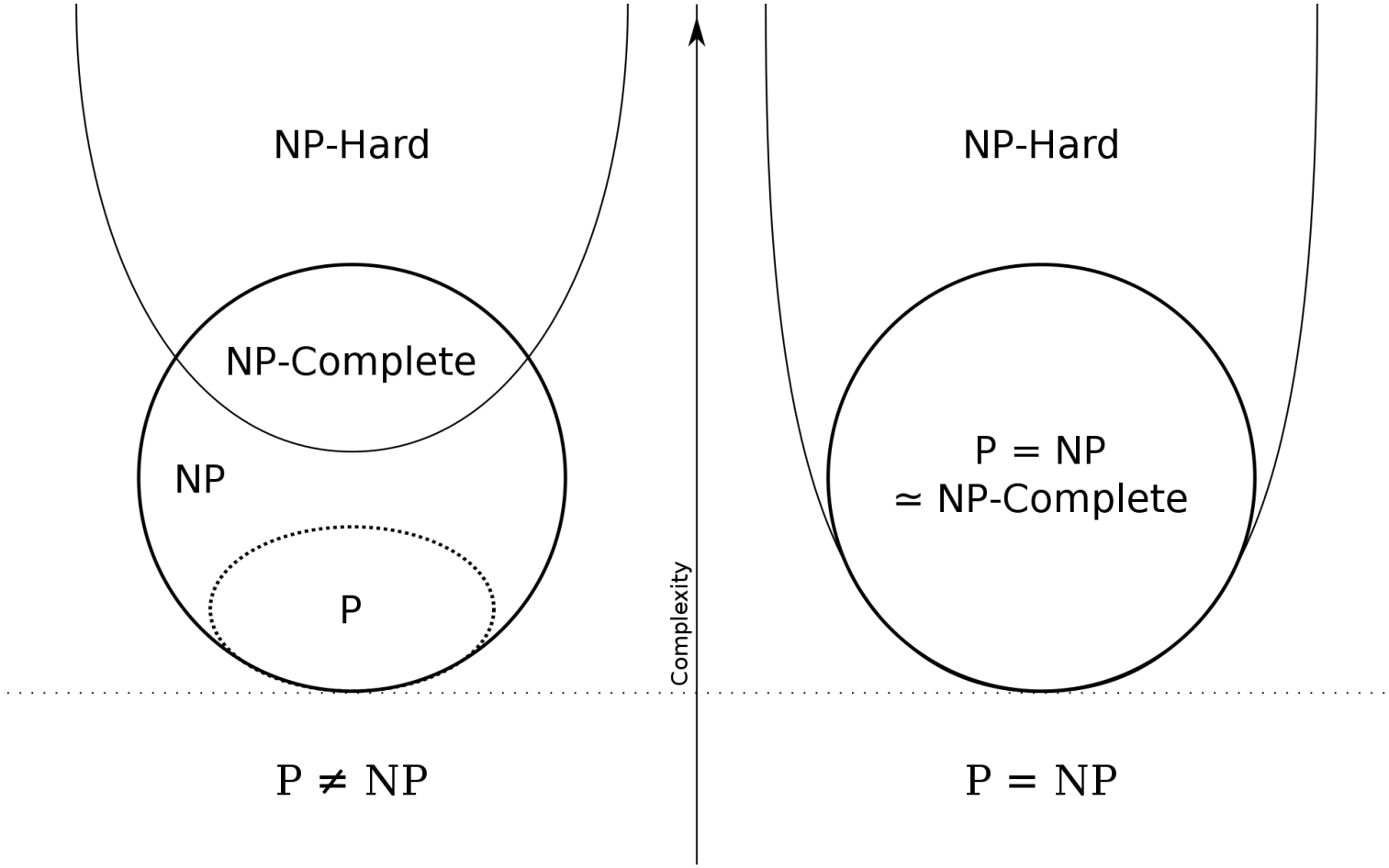



- The bound $2\mathrm{maxdeg}(G)$ is often not optimal:




$2 \cdot \mathrm{maxdeg}(G) = 4$

# The dimension problem is hard




- Finding $dim(G)$ is known to be NP hard
- In everyday language, finding $dim(G)$ can get arbitrary hard

**Thank you for your attention!**

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I hope that was of some help.