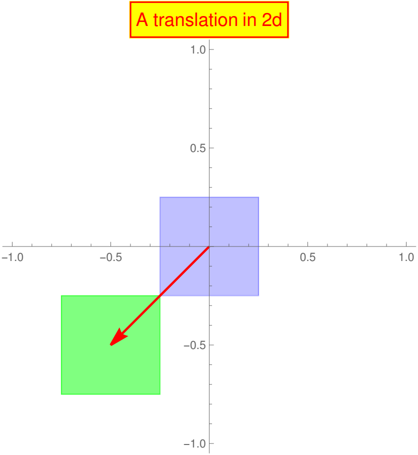
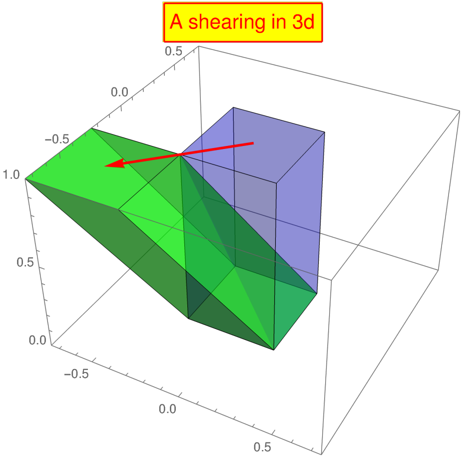
What is...an affine map?

Or: Translations in action.

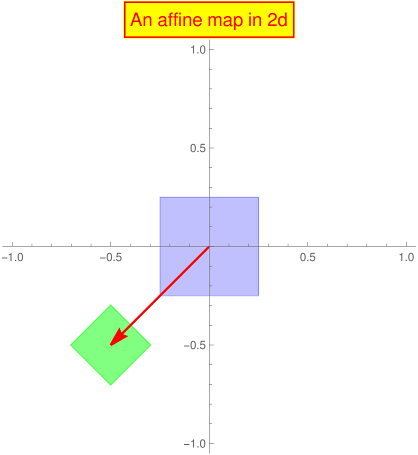
Translation and shearing

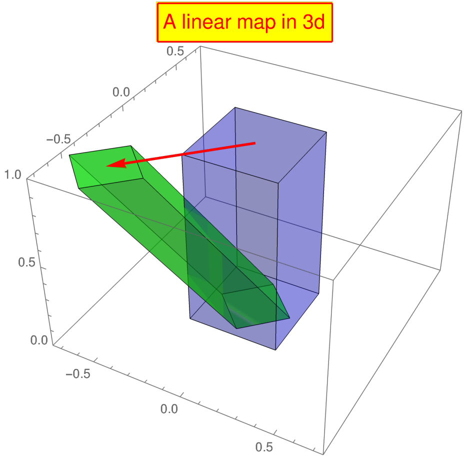




A translation in 2d “is” a shearing in 3d

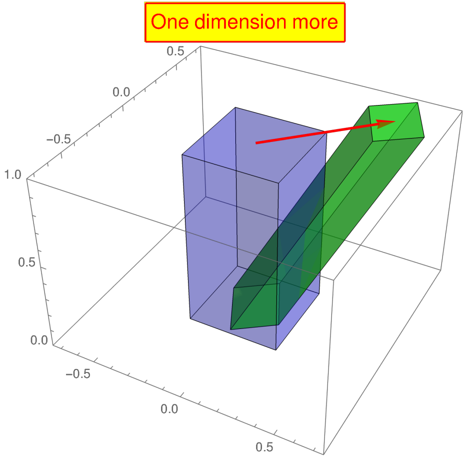
Affine and linear





Rotate-shrink-translate in 2d “is” rotate-shrink in 3d

Affine matrices



$$\leftrightsquigarrow \left(\begin{array}{cc|c} 0.4 & -0.3 & 0.5 \\ 0.3 & 0.6 & 0.5 \\ \hline 0 & 0 & 1 \end{array}\right)$$

Just look one dimension higher!

For completeness: A formal definition.

Let $(A, V), (B, W)$ be affine spaces over a field $\mathbb{K}$. An affine map $(f, m_f)\colon (A, V) \to (B, W)$ is a pair such that:

(a) $f\colon A \to B$ is a map

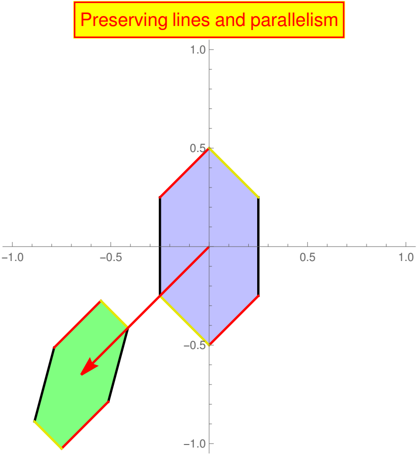
(b) $m_f\colon V \to W$ is a linear map

(c) We have $f(x) - f(y) = m_f(x - y)$

Matrix representation a.k.a. augmented matrix

$$(f, m_f) \leftrightsquigarrow f(x) = m_f(x) + b \leftrightsquigarrow \left(\begin{array}{c|c} M & b \\ \hline 0 & 1 \end{array}\right)$$

Affine transformations preserve lines and parallelism



This is basically why they were defined

Thank you for your attention!

I hope that was of some help.