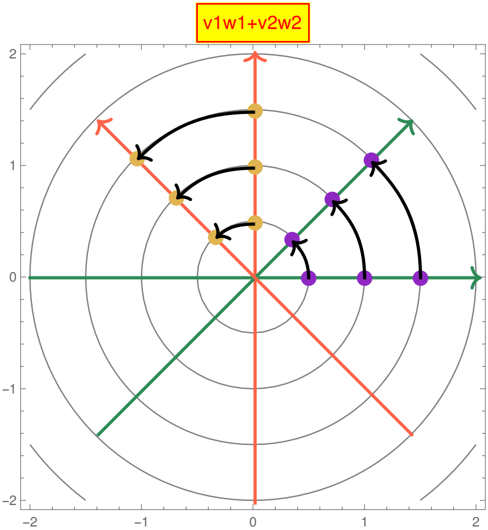
What is...orthogonality?

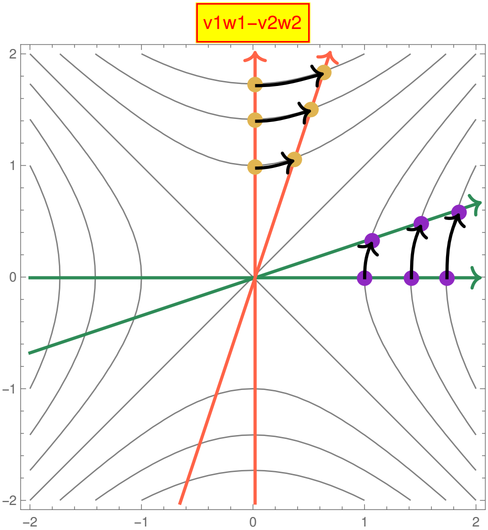
Or: Flavors of 90 degrees.

The Euclidean example: $\langle v, w\rangle = v_1 w_1 + v_2 w_2$



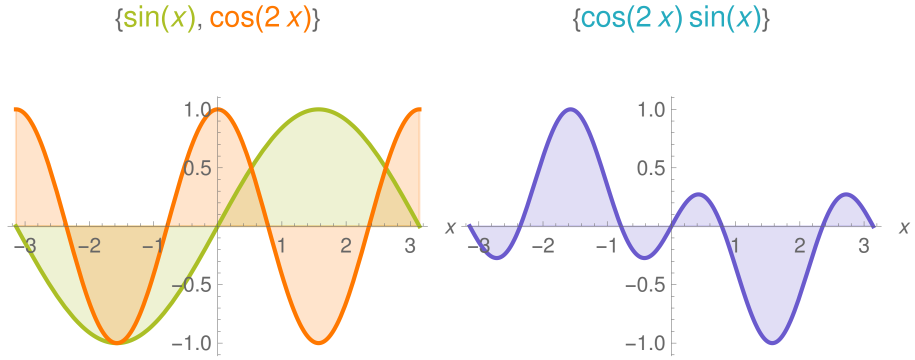
Orthogonality $v \perp w \Leftrightarrow \langle v, w\rangle = 0$ "moves" along circles

The hyperbolic example: $\langle v, w \rangle = v_1 w_1 - v_2 w_2$



Orthogonality $v \perp w \Leftrightarrow \langle v, w \rangle = 0$ “moves” along hyperbolas

A function example: $\langle f, g\rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$



Orthogonality $v \perp w \Leftrightarrow \langle v, w\rangle = 0$ means vanishing of the area underneath

For completeness: A formal definition.

A version of an inner product is a map

$$\langle \_, \_ \rangle \colon V \times V \to \mathbb{R}$$

satisfying linearity, symmetry and maybe:

(a) $\langle v, v \rangle > 0$ for $v \neq 0$ (Positive definite)

(b) $\langle v, v \rangle \geq 0$ for $v \neq 0$ (Positive semi-definite)

(c) No condition on $\langle v, v \rangle$ (No name)

Orthogonality with respect to $\langle \_, \_ \rangle$:

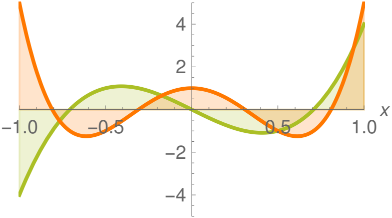
$$v \perp w \Leftrightarrow \langle v, w \rangle = 0$$

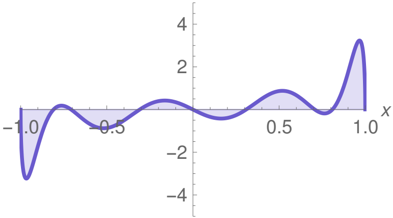
This still satisfies plenty of properties we are used using 90°!

The Chebyshev polynomials $U_{n+1}(x) = 2xU_n(x) - U_{n-1}(x)$

$\left\{8x^3 - 4x,\ 16x^4 - 12x^2 + 1,\ \sqrt{1 - x^2}\right\}$

$\left\{\sqrt{1 - x^2}\,(8x^3 - 4x)(16x^4 - 12x^2 + 1)\right\}$





Chebyshev polynomials play an important role in mathematics – they are Fibonacci-type polynomials – also because of their orthogonality properties

Thank you for your attention!

I hope that was of some help.